Chapter 1
Relations & Functions

Consider the sets A — {1,2,3,4,5} and B— {3,4,5,6,7}. The Cartesian product of A and B is A x B — {(1,3), (1,4), (1,5), (1,6), (1,7), (2,3), (5,6), (5,7)}.

A subset of A ^{x} B by introducing a relation R between the first element 'x' and the second element 'y' of each ordered pair (x, y) as

R— {(x,y): x is greater than y, x e A, y e B}. Then R— {(4,3), (5,3), 5,4)}.

Note]: Relation R from a non-empty set A to a non-empty set B is a subset o/ the Cartesian product A x B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A x B. The second element is called the image oftheJirst element.

Note2: The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

Note3: The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the co-domain of the relation R. Note that range co-domain.

Tips ☞

1. A relation may be represented algebraically either by Roster method or by Set-builder method.
2. An arrow diagram is a visual representation of a relation.
3. The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A x B. If n(A) = p and n(B) q, then n (A ^{x} B) pq and the total number of relations is 2^{q}.
4. A relation R from A to A is also stated as a relation on A.

Inverse relation: If D {(a,b) : a, be R is a relation from set A to a set B, then inverse of ^{R}= R-1 = { (b,a) : a, beR}.

Note: Domain(R) = Range(R) and Range(R) = Domain R^{-1} l.

Types of relations

A relation R in a set A to itself is called:

1. Universal relation: If each element of A is related to every element of A. i.e., R= A × A
2. An identity relation if R {(a,a) : a GA}
3. An empty or void relation if no element of A is related to any element of A. i.e.,

   Note: Empty relation and the universal relation are sometimes called trivial relations. R = A

4. A relation R in a set A is said to be

   a) Reflexive, if every element of A is related to itself. V a GA. i.e., a Ra V aeA.

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b) Symmetric, if then V i.e., a Rb = bRa V a,beR.

c) Transitive, if and $(b,c) \in R \Rightarrow (a,c) \in R$ V a,b,ceR. i.e., a RID and [DRc aRc

5. Equivalence Relation: A relation R in a set A is called an equivalent if

i) R is reflexive, ii) R is symmetric and iii) Ris transitive.

Note: l. If R and S are two relations on a set A, then RAS is also an equivalence relation on A.

2. The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.
3. The inverse of an equivalence relation is an equivalence relation.

Functions: Let A and B be two non-empty sets. A function f from A to B is a correspondence which associates elements of set A to element of set B such that

i. all elements of set A are associated to elements in set B.

ii. an element of set A is associated to a unique element in set B.

Iffis a function from A to B and (a, b) G j; then f(a)=b, where 'b' is called the image of 'a' under fand 'a' is called the pre-image of 'b' under J.'

The functionffrom A to B is denoted by Ji A → B.

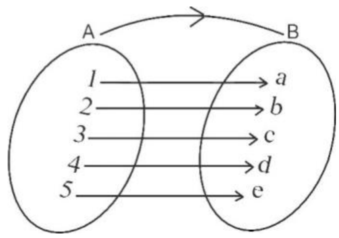
Types of Functions

One-one function (Injective): A function f: A —5B is said to be an one-one function if different elements of A have different images in B.

No. of one-one functions from A to B – – I ^{n}Pm, if n 2 m

1 0 if n = 0

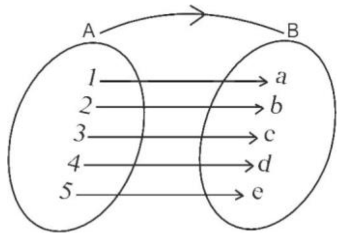
To check the injectivity of a function



i. Take two arbitrary elements x I and x2 in the domain of f. ii. Check whether f - f) iii. If $f(x_1) = f(x_2)$, which implies that = x2 only then the function is a one-one function or injective function otherwise not.

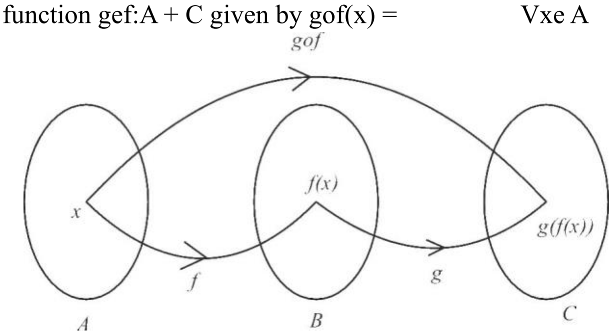
Onto function (surjective): A function f : A •B is said to be an onto function, if every element of B is the image of some element of A under n i.e., for every element of yeB , there exists an element x GA such that f (x) = y .

One-one onto function (bijective): A function f : A •B is said to be an one-one and onto, if it is both one-one and onto.



Composition of function

Let f : A B and g : A B be any two functions, the composition of f and g , denoted by gof is defined as the function gef:A + C given by gof(x) = Vxe A



Invertible function

Let f : A and g: A + B be any two functions, the composition of / and g , denoted by gof is defined as the function $gof : A \to C$ given by gof(x) $= g\left[f(x)\right]$ VxeA. For example, Let f be given by f (x) =4x+3. Show that f (x) is invertible. Also find the inverse off.•

$f(x) = 4x + 3$

f(X1) +3

$f(x_2$ — 4X2 + 3

$f(x_1) = f(x_2) \Rightarrow 4x_1$ + 3 4x2 + 3 4Xl — —4X2= x_1 =

∴ f is one—one.

Again, y 4x+ 3

$y - 3 = 4x \Rightarrow x = \frac{y-3}{4}$

$f(x) = f$ ⌈v-3h $= 4 \times \frac{y-}{4}$ +3 =y-3+3= Y is onto.

Hence, f is one-one, onto and therefore, invertible.

NOW, Y f (X) = y = $4x$ +3 X= $\frac{y-3}{4} \Rightarrow f^{-1}(y) = \frac{y-3}{4}$

Binary operation

An operation * on a non-empty set A, satisfying the closure property is known as a binary operation. For example, let * be the binary operation on N given by a LCM of a and b. Find

l. 3 = LCM of4 and 3= $4 \times 3 =$

11. 24 LCM of 16 and 24= $2 \times 2 \times 2 \times 2 \times 3 = 48$

Properties of binary operation:

l. Commutative Property: A binary operation * on set A is said to be commutative, if $a*b=b*a,$ for alla,b e A.

2. Associative Property: A binary operation * on set A is said to be associative, if (a = b* c), for al/ e A.

3. Identity property: A binary operation * on set A is said to be identity, if an element e G A, if $a*e=a=e*a,\forall$a GA.

For example, find the identity element in Z for * on Z, defined by a *b —a+b+l. Let e be the identity element in Z.

a*e=a

∴ a*b=a*e=a+e+l

• —l e Z is the identity element for * .

Write the operation table of * on the set 1,2,3,4,5 defined by a *b = $\min\{a,b\}$.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

Tips ☞

1. $(fog)(x) = f(g(x))$
2. $(fof)(x) = f(f(x))$
3. $(gof)(x) = g(f(x))$
4. $(gog)(x) = g(g(x))$
5. $(fof^{-1})(x) = f(f^{-1}$, etc..