Chapter 6

Electromagnetic Induction

6.1 Introduction

In the early decades of the nineteenth century, experiments on electric current by Oersted, Ampere and a few others established the fact that electricity and magnetism are inter-related. They found that moving electric charges produce magnetic fields.

the converse effect possible?

The experiments of Michael Faraday in England and Joseph Henry in USA, demonstrated that electric currents were induced in closed coils when subjected to changing magnetic fields. The pioneering experiments of Faraday and Henry have led directly to the development of modern day generators and transformers.

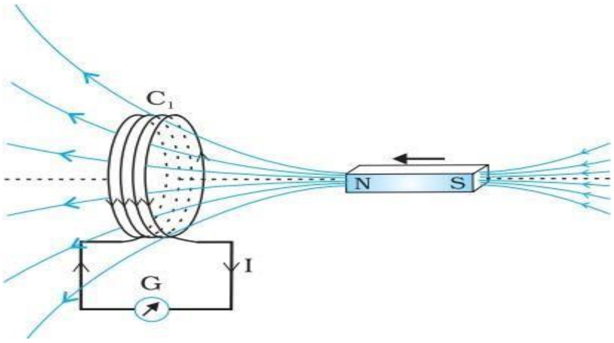
Electromagnetic Induction

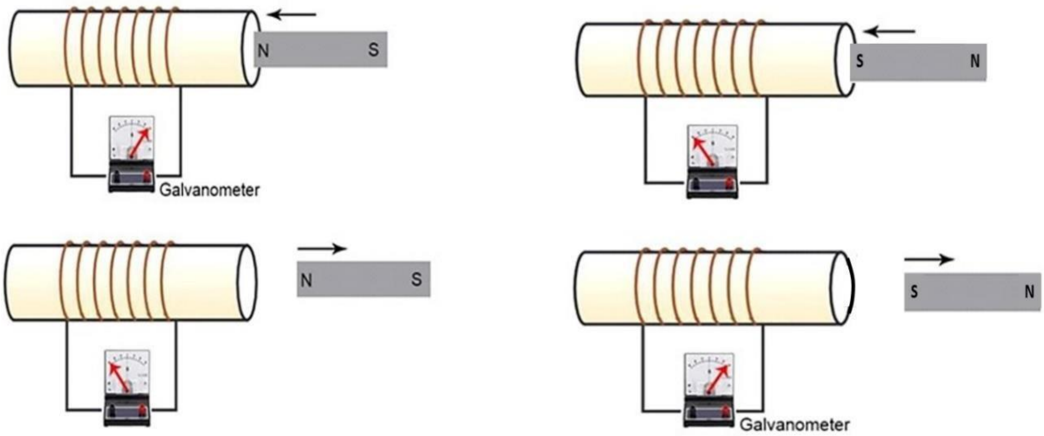
The phenomenon in which electric current is generated by varying magnetic fields is appropriately called electromagnetic induction.

6.2 The Experiments of Faraday and Henry

Experiment 1

A coil C_1 is connected to a galvanometer G.





Observations

- When the North-pole of a bar magnet is pushed towards the coil, the pointer in the galvanometer deflects, indicating the presence of electric current in the coil.
- The deflection lasts as long as the bar magnet is in motion.

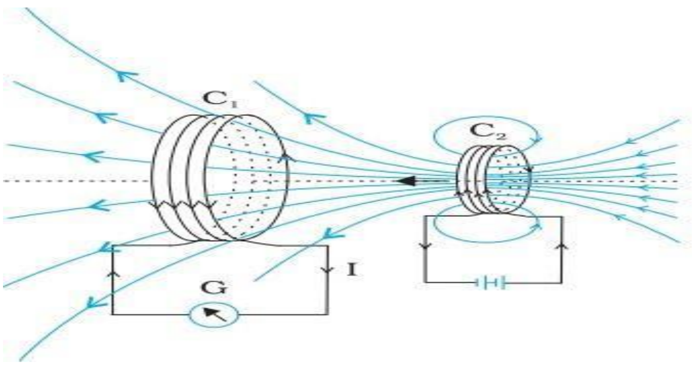
- The galvanometer does not show any deflection when the magnet is held stationary.
- When the magnet is pulled away from the coil, the galvanometer shows deflection in the opposite direction, which indicates reversal of the current's direction.
- Moreover, when the South-pole of the bar magnet is moved towards or away from the coil, the deflections in the galvanometer are opposite to that observed with the North-pole.
- Further, the deflection (and hence current) is found to be larger when the magnet is pushed towards or pulled away from the coil faster.
- When the bar magnet is held fixed and the coil C_1 is moved towards or away from the magnet, the same effects are observed.

Conclusion

It shows that it is the relative motion between the magnet and the coil that is responsible for generation (induction) of electric current in the coil.

Experiment 2

The bar magnet is replaced by a second coil C_2 connected to a battery.
The steady current in the coil C_2 produces a steady magnetic field.



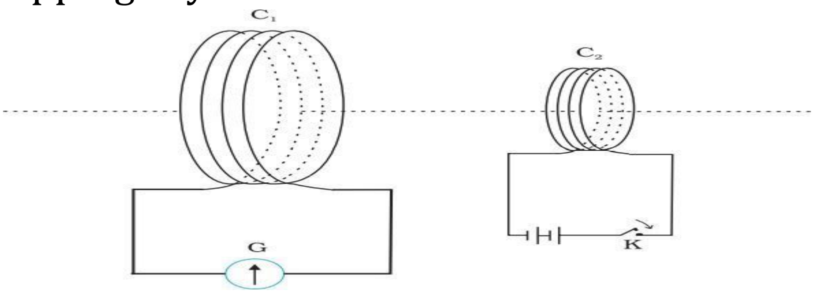
Observations

- As coil C_2 is moved towards the coil C_1, the galvanometer shows a deflection. This indicates that electric current is induced in coilC_2.
- When C_2 is moved away, the galvanometer shows a deflection in the opposite direction.
- The deflection lasts as long as coil C_2 is in motion.
- When the coil C_2 is held fixed and C_1 is moved, the same effects are observed.

Again, it is the relative motion between the coils that induces the electric current.

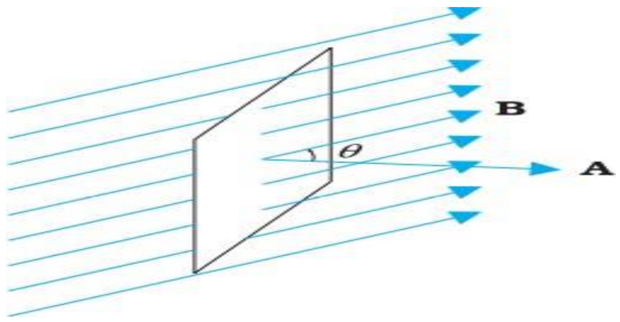
Experiment 3

Two coils C_1 and C_2 are held stationary. Coil C_1 is connected to galvanometer G while the second coil C_2 is connected to a battery through a tapping key K.



- The galvanometer shows a momentary deflection when the tapping key K is pressed. The pointer in the galvanometer returns to zero immediately.
- If the key is held pressed continuously, there is no deflection in the galvanometer.
- When the key is released, a momentory deflection is observed again, but in the opposite direction.
- It is also observed that the deflection increases dramatically when an iron rod is inserted into the coils along their axis.

6.3 Magnetic Flux



Magnetic flux through a plane of area A placed in a uniform magnetic field B can be written as

$\phi_B = B . A = BA \cos \theta$ where θ is angle between B and A.

The SI unit of magnetic flux is weber(Wb) or tesla meter squared(T m^2).

Magnetic flux is a scalar quantity.

The flux can be varied by changing any one or more of the terms B, A and θ .

6.4 Faraday's law of electromagnetic induction

The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit.

$$\varepsilon = -\frac{d\phi_B}{dt}$$

The negative sign indicates the direction of ε and hence the direction of current in a closed loop.

In the case of a closely wound coil of N turns, the total induced emf ,

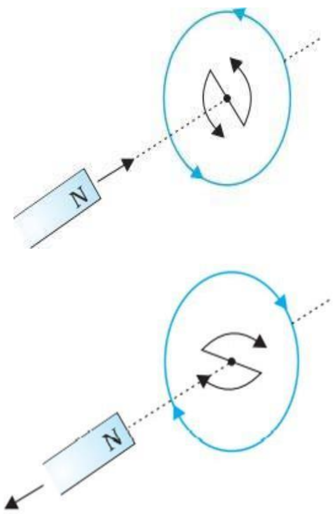
$$\varepsilon = -N\frac{d\phi_B}{dt}$$

The induced emf can be increased by increasing the number of turns N of a closed coil.

Lenz's Law

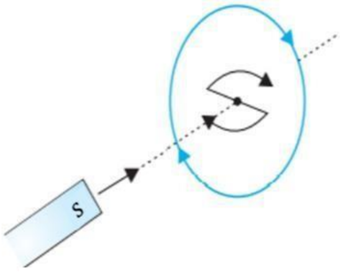
German physicist Heinrich Friedrich Lenz deduced a rule, known as Lenz's law which gives the polarity of the induced emf .

The statement of the law is: The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it.

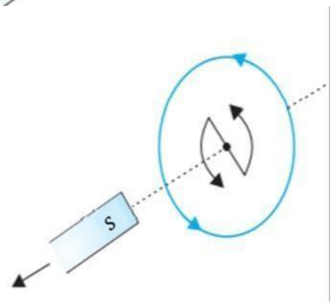


When North-pole of a bar magnet is moved towards the coil, a current is induced in the coil in such a direction that it opposes the increase in flux. That means the face of coil (towards magnet) should have North-polarity .So current in that face will be anti clockwise (counter- clockwise) .

When North-pole of a bar magnet is moved away from the coil, a current is induced in the coil in such a direction that it opposes the decrease in flux. That means the face of coil (towards magnet) should have South-polarity.So current in that face will be clockwise.



When South-pole of a bar magnet is moved towards the coil, the face of coil (towards magnet) should have South-polarity. So current in that face will be clockwise.



When South-pole of a bar magnet is moved away from the coil, the face of coil (towards magnet) should have North-polarity .So current in that face will be anti clockwise (counter- clockwise) .

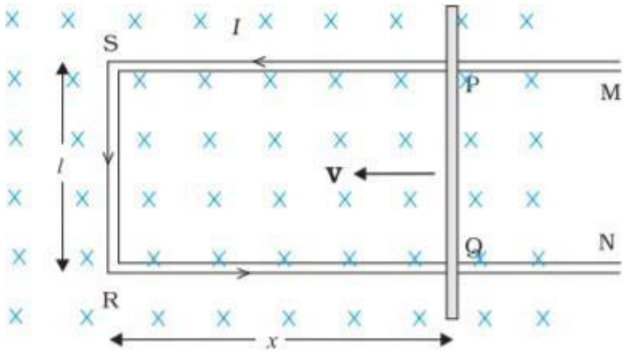
6.5 Lenz's Law and Conservation of Energy

If the induced current was in the same direction changing magnetic flux, the front face of coil gets South polarity ,when the north pole of bar magnet is pushed into the coil .The bar magnet will then be attracted towards the coil at a increasing acceleration and kinetic energy will continuously increase without expending any energy. This violates the law of conservation of energy and hence can not happen.

The current induced in the coil is opposite to the direction of changing magnetic flux. Then the bar magnet experiences a repulsive force due to the induced current. Therefore, a person has to do work in moving the magnet. This energy(work) is dissipated by Joule heating produced by the induced current.Thus Lenz's law is in accordance with law of conservation of energy.

6.6 Motional Electromotive Force

When a conducting rod is moved through a constant magnetic field, an emf is developed between the ends of the rod. This emf is known as Motional Emf.



Consider a straight conductor moving in a uniform and time independent magnetic field.The magnetic flux Φ enclosed by the loop PQRS, $\phi = Blx$

Since x is changing with time, the rate of change of flux Φ will induce an emf given by:

$$\varepsilon = -\frac{d\phi_B}{dt} = \frac{d}{dt}(Blx)$$

$$\varepsilon = -Bl\frac{dx}{dt}$$

$$v = -\frac{dx}{dt} \text{ is the speed of the conductor}$$

$$s = \boldsymbol{Blv}$$

The induced emf Blv is called motional emf.

6.7 Inductance

An electric current can be induced in a coil by flux change produced by the same coil or a flux change produced by a neighbouring coil .These phenomenon are respectively called self induction and mutual induction.

In both the cases, the flux through a coil is proportional to the current.

$$\phi \propto I$$

$$\boldsymbol{\phi} = L\,I$$

The constant of proportionality, in this relation, is called inductance.
Inductance is a scalar quantity. It has the dimensions of $[M L^2 T^{-2} A^{-2}]$. The SI unit of inductance is henry and is denoted by H

Self-Induction

The phenomenon of production of induced emf in an isolated coil by varying current through the same coil is called self-induction.
The flux linked with the coil is proportional to the current through the coil.

$$\phi \alpha I$$

$$\phi = L I \text{ ------------- } (1)$$

where constant of proportionality L is called self-inductance of the coil. It is also called the coefficient of self-induction of the coil.
When the current is varied, the flux linked with the coil also changes and an emf is induced in the coil.

For N turns,

$$\varepsilon = -\frac{d\phi}{dt}$$

$$\varepsilon = -\frac{dLI}{dt}$$

$$\varepsilon = -L\frac{dI}{dt} \text{ -----------} (2)$$

Thus, the self-induced emf always opposes any change (increase or decrease) of current in the coil.

Self-Inductance of a Long Solenoid

Consider a solenoid of cross sectional area A and length l, having n turns per unit length.

The total flux linked with N turns of the solenoid ,

$$\phi = NB\,A$$

$$B = \mu_0 n I$$

$$N = nl$$

$$\phi = nl\,(\mu_0 n I)\,A$$

$$\phi = \mu_0 n^2 A l I \text{ ------------- } (1)$$

$$\text{But, } \phi = LI \text{ ----------------- } (2)$$

From eq (1) and (2)

$$LI = \mu_0 n^2 A l I$$

$$L = \mu_0 n^2 A l \text{ ------------- } (3)$$

If we fill the inside of the solenoid with a material of relative permeability μ_r (for example soft iron, which has a high value of relative permiability), then,

$$L = \mu_r \mu_0 n^2 A l \text{ ---------- } (4)$$

The self-inductance depends on geometry of coil and on the permeability of the medium.

Back emf

The self-induced emf is also called the back emf as it opposes any change in the current in a circuit. Physically, the self-inductance plays the role of inertia. Self inductance is the electromagnetic analogue of mass in mechanics. So, work needs to be done against the back emf (ε) in establishing the current. This work done is stored as magnetic potential energy.

Energy stored in an inductor

W=qV

W=It|ε|

$dW = |\varepsilon| I\,dt$, $\frac{dW}{dt} = |\varepsilon| I$

$$\frac{dW}{dt} = |\varepsilon| I$$

$$\text{But, } |\varepsilon| = L\frac{dI}{dt}$$

dW dI

$$\_\_ = LI \,\_$$

$$dt \quad dt \quad I \qquad dI$$

$$W = \int dW = \int LI \quad 0$$

$$dt$$

$$W = \frac{1}{2} LI^2$$

This expression is analogous to $\frac{1}{2} mv^2$, kinetic energy of a particle of mass m, and shows that L is analogus to m (i.e., L is electrical inertia and opposes growth and decay of current in the circuit).

Mutual induction

The phenomenon of production of induced emf in a coil by varying the current through a neighbouring coil is called mutual-induction.

The flux linked with the coil is proportional to the current through the neighbouring coil. $\phi \propto I$

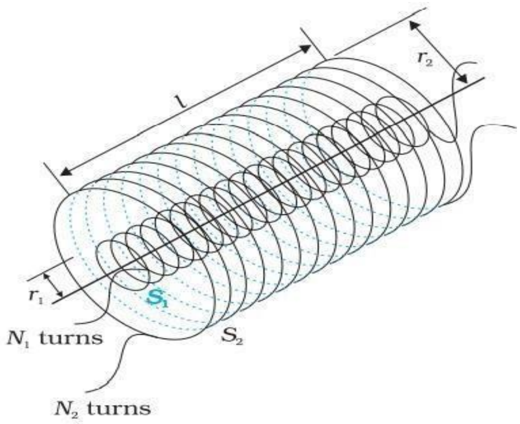
$$\phi = MI$$

where constant of proportionality M is called mutual-inductance of the coil. It is also called the coefficient of mutual-induction of the coil.

When the current in the nighbouring coil is varied, the flux linked with the first coil changes and an emf is induced in the coil.

$$\varepsilon = -M\frac{dI}{dt}$$

Mutual inductance of two co-axial solenoids



Two long co-axial solenoids of same length l.
Inner solenoid S_1 of radius r_1 and the number of turns per unit length n_1.
Outer solenoid S_2 of radius r_2 and the number of turns per unit length n_2.

The current I_2 in S_2 sets up a magnetic flux in S_1.

$$\phi_1 = N_1 B_2 A_1$$

$$B_2 = \mu_0 n_2 I_2$$
$$N_1 = n_1 l$$

$$\phi_1 = (n_1 l)(\mu_0 n_2 I_2) A_1$$

$$\phi_1 = \mu_0 n_1 n_2 A_1 l I_2 \text{-------------}(1)$$

But, $\phi_1 = M_{12} I_2$ ----------------------- (2)

From eq(1) and (2)

$$M_{12} I_2 = \mu_0 n_1 n_2 A_1 l I_2$$

$$\boldsymbol{M_{12} = \mu_0 n_1 n_2 A_1 l}$$

Similarly we get

$$\boldsymbol{M_{21} = \mu_0 n_1 n_2 A_1 \; l} \qquad \text{where } A_1 = \pi r_1^2$$

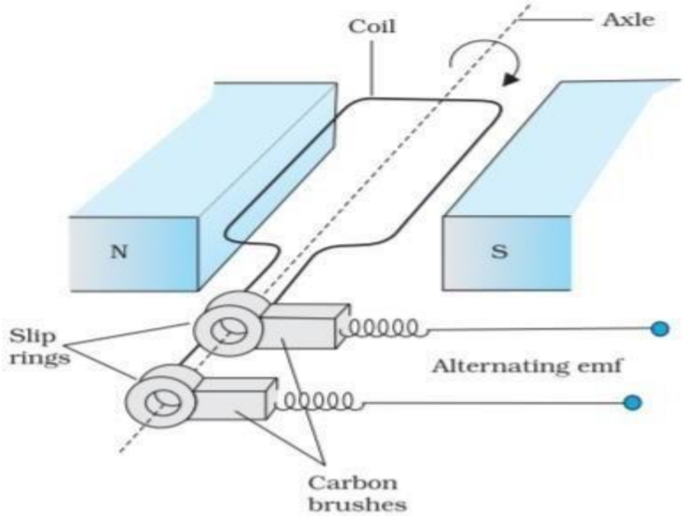
(The flux due to the current I_1 in S_1 can be assumed to be confined solely inside S_1 since the solenoids are very long. So we can take A= πr_1^2 itself)

$$\boldsymbol{M_{12} = M_{21} = M}$$

If a medium of relative permeability μ_r is introduced inside the solenoid

$$M = \mu_r \mu_0 n_1 n_2 A_1 l$$

66.8 AC Generator



An ac generator converts mechanical energy into electrical energy. It consists of a coil which is mechanically rotated in the uniform magnetic field by some external means. The rotation of the coil causes the magnetic flux through it to change, so an emf is induced in the coil.

The magnetic flux at any time t is $\phi = BA \cos \theta$
$= BA \cos \omega t$

From Faraday's law, the induced emf for the rotating coil of N turns is

$$\varepsilon = -N \frac{d\phi}{dt}$$

$$\varepsilon = -N \frac{d}{dt} BA \cos \omega t$$

$$\varepsilon = -NBA \frac{d}{dt} \cos \omega t$$

$$\varepsilon = NBA\omega \sin\omega t$$

$$\varepsilon = \varepsilon_0 \sin\omega t$$

where $\varepsilon_0 = NBA\omega$ is the maximum value of the emf.

ω = 2πν , ν=frequency of revolution of the generator's coil The direction of the current changes periodically and therefore the current is called alternating current (ac).

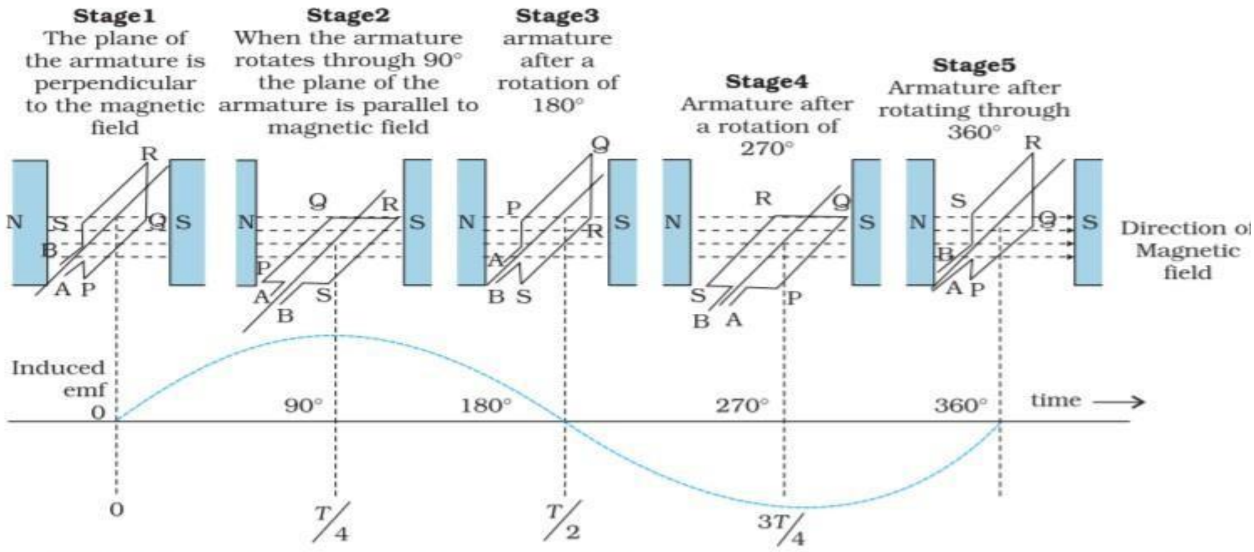


FIGURE 6.17 An alternating emf is generated by a loop of wire rotating in a magnetic field.

Outstanding Guidance for Youth