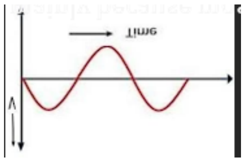
Chapter 7

Alternating Current

7.1 Introduction

The electric mains supply in our homes and offices is a voltage that varies like a sine function with time. Such a voltage is called alternating voltage
(ac voltage) and the current driven by it in a circuit is called

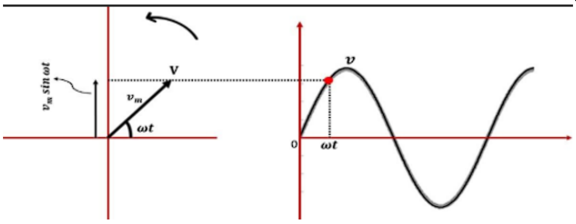
the alternating current (ac current) 

Today, most of the electrical devices we use require ac voltage. This is mainly because most of the electrical energy sold by power companies is transmitted and distributed as alternating current. The main reason for preferring use of ac voltage over dc voltage is that ac voltages can be easily and efficiently converted from one voltage to the other by means of transformers. Further, electrical energy can also be transmitted economically over long distances.

7.2 Representation of ac current and voltage by rotating vectors — phasors

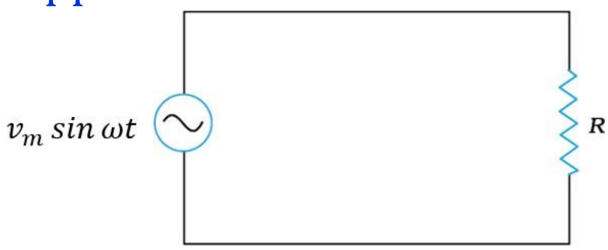
In order to show phase relationship between voltage and current in an AC circuit, we use the notion of phasors.

A phasor is a vector which rotates about the origin in anticlockwise direction with angular speed ω.



- The length of each phasor represents the amplitude or peak value of the voltage or current.
- The projection of each phasor on the vertical axis gives the instantaneous value of the quantity that the phasor represents.
- The rotation angle of each phasor is equal to the phase of alternating quantity at that instant t.
- The angle between two phasors will give you the phase difference between the corresponding quantities

7.3 AC Voltage Applied to a Resistor



Apply Kirchhoff's Loop
rule
,Σε(t)
= 0 v_m
sin

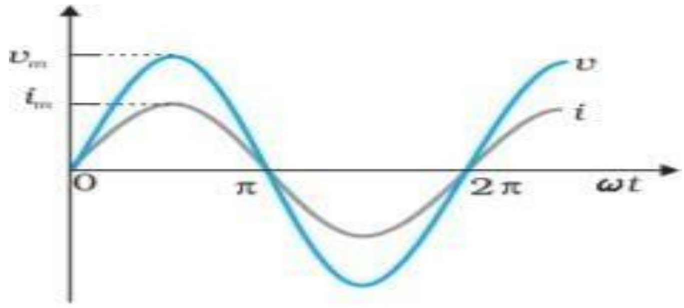
ωt- i R
= 0 vm
sin
ωt=
viRm

i = sin ωt
R

$i = i_m \sin \omega t$ where **$i_m = \frac{v_m}{R}$**

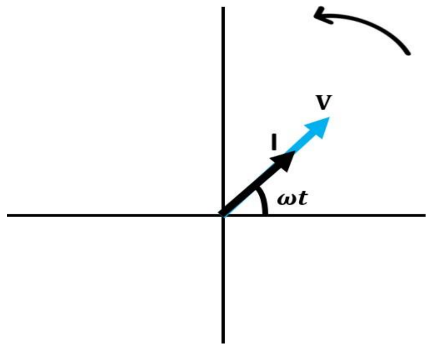
i_m is called amplitude of current

Graph of voltage and current across a pure resistor versus ωt



In a pure resistor, the voltage and current are in phase. The minima, zero and maxima occur at the same.

Phasor diagram for the circuit



Power Dissipated in the Resistor

The ac current varies sinusoidally and has corresponding positive and negative values during each cycle. Thus, the sum of the instantaneous current values over one complete cycle is zero, and the average current is

zero.The fact that the average current is zero, however, does not mean that the average power consumed is zero and that there is no dissipation of electrical energy.
The instantaneous power dissipated in

the resistor i

$p = vi$

$p = v_m \sin \omega t$
$i_m \sin \omega t$ p
$= v_m i_m$
$\sin^2 \omega t$

Avearage power consumed over one complete cycle

$\bar{p} = \; < v_m i_m \sin^2 \omega t >$

$\bar{p} = v_m i_m < \sin^2 \omega t >$

$< sn^2 \omega t > = \frac{1}{2}$

$= \frac{1}{2} v \; \bar{p} \; {}_m i_m$

$p = \left(\frac{v_m}{\sqrt{2}}\right) \left(\frac{i_m}{\sqrt{2}}\right)$

P = VI

Where **I or I_{rms}** is called rms current and V **or V_{rms}** is called rms voltage.

The rms current (Root Mean Square Current) or Effective Current

To[1] express AC power $\bar{p} = \frac{1}{2} \; {}_m \; {}_m v \quad i$ in the same form as dc power $P =$
VI , 2
a special value of current is defined and used. It is called, root mean square (rms) or effective current and is denoted by **I_{rms}**or I.

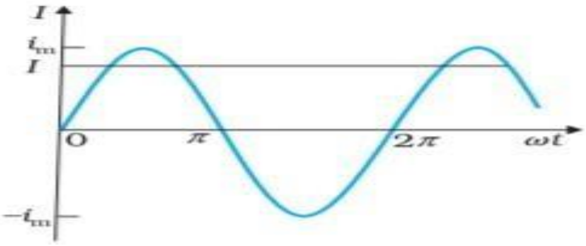
$$I = \sqrt{\langle i^2 \rangle}$$

$$I = \sqrt{\langle (i_m \sin \omega t)^2 \rangle}$$

$$I = i_m \sqrt{\langle \sin^2 \omega t \rangle}$$

$$I = i_m \sqrt{\frac{1}{2}}$$

$$I = \frac{i_m}{\sqrt{2}} = 0.707\, i_m$$



The rms current is the equivalent dc current that would produce the same average power loss as the alternating current.

Similarly ,rms voltage or Effective voltage

$$V = \mathbf{V_{rms}} = \frac{\mathbf{v_m}}{\sqrt{2}}$$

$$V = \frac{v_m}{\sqrt{2}} = 0.707\, v_m$$

Why a shock from 220V ac is more fatal than that from 220Vdc?

The household line voltage of 220 V is an rms value.

$$V = 220V$$

Its peak voltage $v_m = \sqrt{2}\, V$

= 1.414 x 220 V

= 311 V

At some instant peak value of ac may reach upto 311V .So a shock from 220V ac is more fatal than that from 220Vdc.

Example

A light bulb is rated at 100W for a 220 V supply. Find

(a) the resistance of the bulb (b) the peak voltage of the source

(c) the rms current through the bulb.

(a) We are given P = 100 W and V = 220 V. The resistance of the bulb is

$$R = \frac{V^2}{P} = \frac{(220\,V)^2}{100\,W} = 484\Omega$$

(b) The peak voltage of the source is

$$v_m = \sqrt{2}V = 311\,V$$

(c) Since, $P = I\,V$

$$I = \frac{P}{V} = \frac{100\,W}{220\,V} = 0.450A$$

7.4 AC Voltage Applied to an Inductor

Apply Kirchhoff's

Loop rule ,Σε(t)

= 0 di v sin ωt-

L __ =0 m dt

di v sin ωt= L

$$\frac{di}{dt} = \frac{v_m}{L}\sin\omega t$$

$$di = \frac{v_m}{L}\sin\omega t\, dt$$

$$i = \frac{v_m}{L}\int \sin\omega t\, dt$$

$$i = \frac{v_m}{L} \times \frac{-\cos\omega t}{\omega}$$

$\cos\theta = \sin(90-\theta)$
$-\cos\theta = -\sin(90-\theta)$
$-\sin\theta = \sin(-\theta)$
$-\cos\theta = \sin(-90+\theta)$
$-\cos\theta = \sin(\theta-90)$
$-\cos\omega t = \sin\left(\omega t - \frac{\pi}{2}\right)$

$$i = -\frac{v_m}{\omega L}\cos\omega t$$

$$i = -i_m\cos\omega t$$

$$\mathbf{i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)} \text{ where } \mathbf{i_m = \frac{v_m}{\omega L}}$$

In a pure inductor, the current lags the voltage by π/2 or one-quarter (1/4) cycle.

Inductive Reactance (X_L)

The current amplitude, $i_m = \frac{v_m}{\omega L}$

$$i =$$

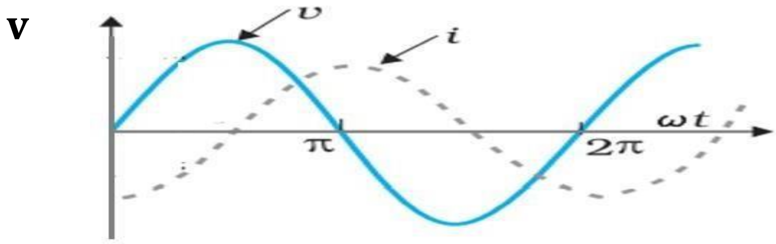
v
m
m
X
L

The quantity ω L is analogous to the resistance and is called inductive reactance, denoted by X_L

$$X_L = \omega L = 2\pi f L$$

- The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω).
- The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit.
- The inductive reactance is directly proportional to the inductance and to the frequency of the current.
- For DC , f=0 and so X_L =0 i.e., an inductor offers an easy path to DC.
- The value of X_L increases as frequency is increased, hence offers a resistive path to AC.

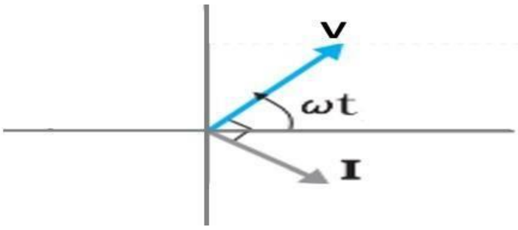
Graph of v and i versus ωt



=
v
m
s
i
n

$$i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Phasor diagram



The current lags the voltage by $\pi/2$.

Power Dissipated in the Inductor

Instantaneous power $p = iv$

$p = -i_m \cos \omega t \times v_m \sin \omega t$

$p = -i_m v_m \cos \omega t \sin \omega t$

$$p = -\frac{i_m v_m}{2} 2\cos \omega t \sin \omega t$$

$$p = -\frac{i_m v_m}{2} \sin(2\omega t)$$

The average power over a complete cycle

$$\bar{p} = P = \left\langle -\frac{i_m v_m}{2} \sin(2\omega t) \right\rangle$$

$P = -\frac{i_m v_m}{2}\langle \sin(2\omega t)\rangle$

$\langle \sin(2\omega t)\rangle = 0$ $P = 0$

The average power supplied to an inductor over one complete cycle is zero.

Example

A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

Inductive reactance, $X_L = \omega L = 2\pi f L$

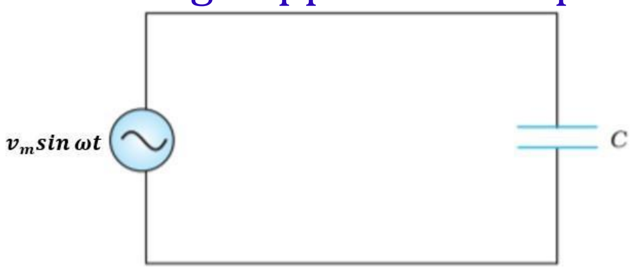
$= 2 \times 3.14 \times 50 \times 25 \times 10^{-3}$

$= 7.85\Omega$

The rms current in the circuit is, $I = \frac{V}{X_L}$

$I = \frac{220}{7.85} = 28A$

7.5 AC Voltage Applied to a Capacitor



Applying Kirchhoff's Loop rule

$\Sigma\varepsilon(t) = 0$

$v_m \sin \omega t - \frac{q}{C} = 0$

s
i
n
ω
t
=
m

C

$q = C\, v_m \sin \omega t$

d

$i = \_$
(
C
v
m
si
n
ω
t)
dt

d

$i = C\, v\, (\sin \omega t)$

—

m dt

$i = C\, v_m\, \omega \cos \omega t$ $i = \omega C\, v_m$
$\cos \omega t$

$i = i_m \cos \omega t$

$\cos\theta = \sin(90 + \theta)$
$\cos\theta = \sin(\theta + 90)$
$\cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$

π i =

$\mathbf{i_m \sin\left(\omega t + \right)}$
2

where $\mathbf{i_m = \omega C\, v_m}$

or $i_m = \frac{v_{1m}}{(\ \ )}$

mC

In a purely capacitive circuit, the current leads the voltage by π/2 or one- quarter (1/4) cycle.

Capacitive Reactance

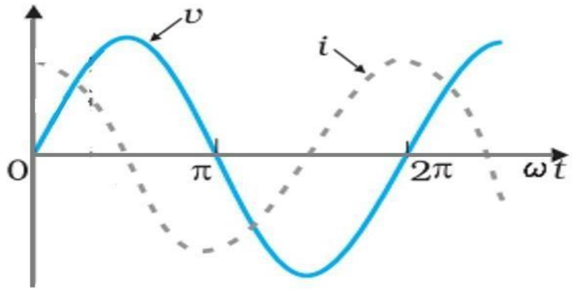
Current amplitude, $i_m = \frac{v_m}{(1/\omega C)} = \frac{v_m}{X_C}$

The quantity ($\frac{1}{\omega C}$) is analogous to the resistance and is called capacitive reactance, denoted by $\mathbf{X_C}$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

- The dimension of capacitive reactance is the same as that of resistance and its SI unit is ohm (Ω).
- The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as the resistance limits the current in a purely resistive circuit.
- Capacitive reactance is inversely proportional to the frequency and the capacitance.
- For DC , f=0 and hence X_C = infinite i.e., the capacitor blocks DC.
- For AC, as the frequency increases, X_C decreases and hence capacitor allows AC to flow through it.

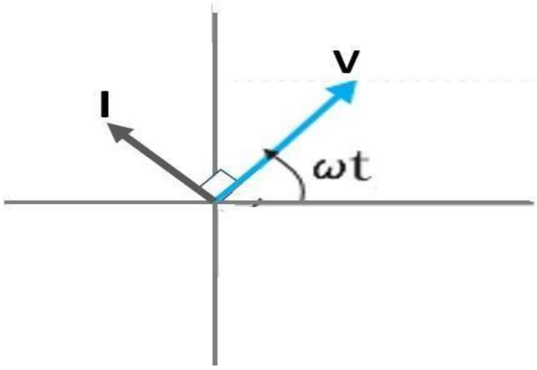
Graph of v and i versus ωt



v = v_m sin ωt $\frac{\pi}{2}$

i = i_m sin (ωt +)

Phasor diagram



Power Dissipated in the Capacitor

P=iv

p= i_m cos ωt x v_m sin ωt

p= $\frac{i_m v_m}{2}$ sin(2ωt)

The average power over a complete cycle $\bar{p}$ = P = ⟨$\frac{i_m v_m}{2}$ sin(2ωt)⟩

$$P = \frac{i_m v_m}{2} \langle \sin(2\omega t) \rangle$$

$\langle \sin(2\omega t) \rangle = 0$ $P = 0$

The average power supplied to a capacitor over one complete cycle is zero.

Example

A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?

The capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$= \frac{1}{2 \text{x} 3.14 \text{x} 50 \text{x} 15 \text{x} 10^{-6}} = 212\Omega$$

The rms current is , $I = \frac{V}{X_C}$

$$I = \frac{220}{212} = 1.04\text{A}$$

The peak current is $i_m = \sqrt{2}\, I$

$= 1.414 \text{ x} 1.04$

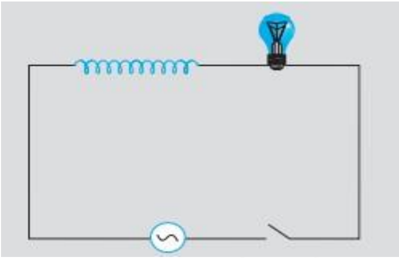
$= 1.47\text{A}$

If the frequency is doubled, the capacitive reactance is halved , and consequently, the current is doubled.

Example

A light bulb and an open coil inductor are connected to an ac source through a key as shown in Figure.

The switch is closed and after sometime, an iron rod is



inserted into the interior of the inductor. The glow of the light bulb

(a)increases; (b) decreases; (c) is unchanged, as the iron rod is inserted.

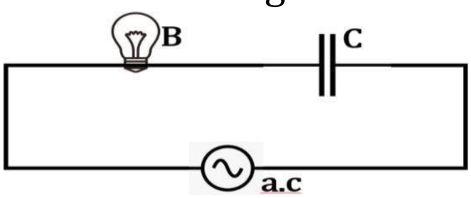
Give your answer with reasons.

Solution:

As the iron rod is inserted, the magnetic field inside the coil magnetizes the iron increasing the magnetic field inside it. Hence, the inductance of the coil increases. Consequently, the inductive reactance of the coil increases. As a result, a larger fraction of the applied ac voltage appears across the inductor, leaving less voltage across the bulb. Therefore, the glow of the light bulb decreases.

Example

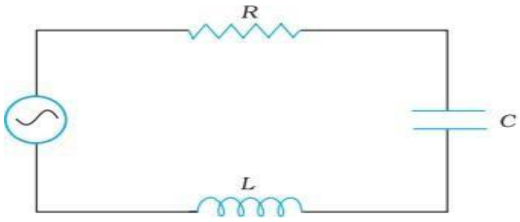
An electric bulb B and a parallel plate capacitor C are connected in series as shown in figure.



The bulb glows with some brightness. How will the glow of the bulb affected on introducing a dielectric slab between the plates of the capacitor? Give reason in support of your answer

When a dielectric slabis introduced between the plates the capacitance increases. Then capacitive reactance decreases. As a result, a smaller fraction of the applied ac voltage appears across the capacitor, leaving large voltage across the bulb. Therefore, the glow of the light bulb increases.

7.6 AC Voltage Applied to a Series LCR Circuit



Applying Kirchhoff's

Loop rule $\Sigma\varepsilon(t) = 0$

$v_m \sin \omega t - iR - L\frac{di}{dt} - \frac{q}{C} = 0$

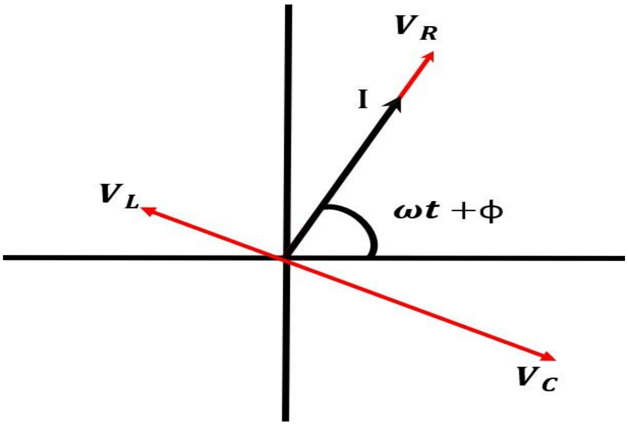
$$v_m \sin \omega t = iR + L\frac{di}{dt} + \frac{q}{C}$$

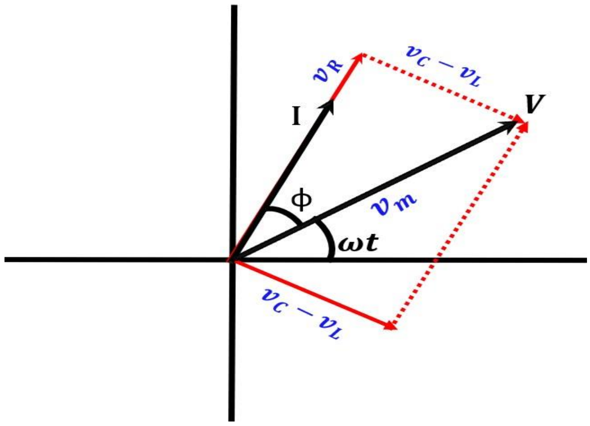
Phasor-diagram solution

Since L,C and R are in series the ac current i in each element is the same. Let the current be $\mathbf{i = i_m \sin(\omega t + \phi)}$

Further, let V_R, V_L, V_C, and V represent the voltage phasors across the resistor, inductor , capacitor and the source, respectively.

For resistor , $\mathbf{V_R}$ and **I are in phase**.
For inductor , $\mathbf{V_L}$ **leads I by** $\pi/2$. For capacitor , $\mathbf{V_C}$ **lags I by** $\pi/2$.





To find the value v_m^2 of= $i v_{mR}^{\,2} + (v_C - v_L)^2$

$$v_m^2 = (i_m R)^2 + (i_m X_C - i_m X_L)^2$$

$$v_m^2 = i_m^2 [(R)^2 + (X_C - X_L)^2]$$

$$i_m^2 = \frac{v_m^2}{(R)^2+(X_C-X_L)^2}$$

$$i_m = \frac{v_m}{\sqrt{(R)^2+(X_C-X_L)^2}}$$

$$\frac{v_m}{i_m} = Z$$

The quantity $\sqrt{(R)^2 + (X_C - X_L)^2}$ is analogous to resistance and is called impedance Z in an ac circuit.

Impedance, $Z = \sqrt{(R)^2 + (X_C - X_L)^2}$

SI unit of Z is Ohm

The phase difference ϕ between voltage and current is ,

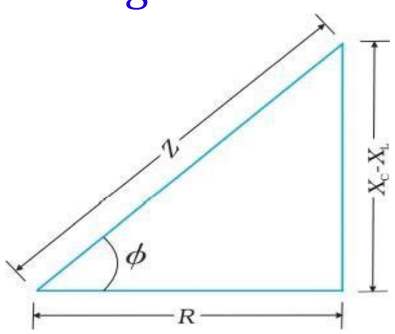
$$\tan\phi = \frac{v_{Cm} - v_{Lm}}{v_{Rm}}$$

$$\tan\phi = \frac{i_m X_C - i_m X_L}{i_m R}$$

$$\tan\phi = \frac{X_C - X_L}{R}$$

$$\boldsymbol{\phi = \tan^{-1} \frac{X_C - X_L}{R}}$$

Impedance diagram



The phase difference ϕ can be obtained using impedance diagram.

$$\tan\phi = \frac{X_C - X_L}{R}$$

$$\phi = \tan^{-1}\frac{X_C - X_L}{R}$$

Example

A resistor of 200 Ω and a capacitor of 15.0 μF are connected in series to a 220 50 Hz ac source.

(a) Calculate the current in the circuit

(b) Calculate the voltage (rms) across the resistor and the capacitor.
Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

a) R = 200Ω , C = 15.0 μF = 15x 10^{-6} F , V = 220 V, **f** = 50Hz

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}$$

$$Z = \sqrt{200^2 + \left(\frac{1}{2x3.14x50x15x\,10^{-6}}\right)^2}$$

$$Z = \sqrt{200^2 + 212.3^2}$$

$$\mathbf{Z} = 291.5\Omega$$

The current in the circuit is

V

$$I = \frac{}{Z}$$

$$\mathbf{I} = \frac{220}{291.5} = 0.755A$$

(b) The current is the same throughout the circuit.

$$V_R = IR = 0.755\ A \times 200\ \Omega = 151V$$

$$V_C = IX_C = 0.755\ A\ x\ 212.3\ \Omega = 160.3V$$

Algebraic sum of V_R and V_C

=151V+160.3V=311.3V This is more than

source voltage and is not possible.

There is a phase difference of 90^0 between V_R and V_C. Therefore, the total of these voltages must be obtained using the Pythagorean theorem.

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{151^2 + 160.3^2} = 220V$$

Resonance

A system oscillating with its natural frequency is driven by an energy source at a frequency that is near the natural frequency, the amplitude of oscillation is found to be large. This phenomenon is called resonance.

A familiar example of this phenomenon is a child on a swing. If the child pulls on the rope at regular intervals and the frequency of the pulls is almost the same as the frequency of swinging, the amplitude of the swinging will be large.

Condition for resonance in an LCR circuit

For an LCR circuit the current amplitude is given by

$$= \frac{v_m}{} = \frac{v_m}{}$$

$i_m \quad Z \quad \sqrt{(R)^2+(X_C-X_L)^2}$

For resonance to happen impedance should be minimum and current maximum. So the condition for resonance is,

$$X_C = X_L$$

Impedance at resonance

$$Z = \sqrt{(R)^2 + (X_C - X_L)^2}$$

$$Z = \sqrt{(R)^2 + 0^2}$$

$$Z = R$$

Impedance is minimum at resonance.

Current Amplitude at Resonance

$$i_m = \frac{v_m}{Z}$$

$$i_{max\,m} = \frac{v_m}{R}$$

Current amplitude is maximum at resonance.

Resonant Frequency

The condition for resonance, $X_C = X_L$

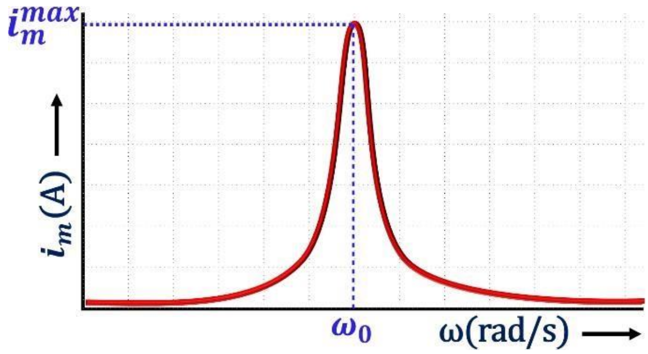
$$\frac{1}{\omega_0 C} = \omega_0 L$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

ω_0 is called Resonant frequency

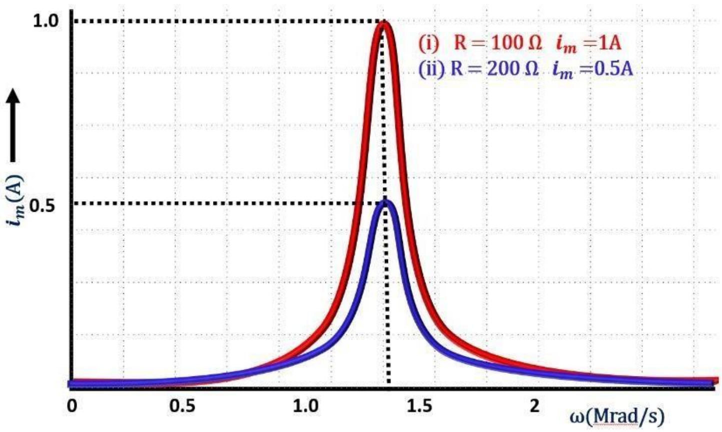
Variation of $\mathbf{i_m}$ with ω



Example

Figure shows the variation of i m with ω in a RLC series circuit with L = 1.00 mH, C = 1.00 nF for two values of R:

(i) R = 100 Ω and (ii) R = 200 Ω. For the source applied vm = 100 V.



For R = 100 Ω

i m = vRm = ___ 100100

= **1**A

For R = 200 Ω

i m = vRm = ___ 100200

= **0. 5**A

Tuning of a radio or TV

Resonant circuits have a variety of applications, for example, in the tuning mechanism of a radio or a TV set. The antenna of a radio accepts signals of dfferent frequencies from many broadcasting stations . But to hear one particular radio station, we tune the radio. In tuning, we vary the capacitance of a capacitor in the tuning circuit such that the resonant frequency of the circuit becomes nearly equal to the frequency of the radio signal received. When this happens, the amplitude of the current with the frequency of the signal of the particular radio station in the circuit is maximum.

Resonance phenomenon is exhibited by a circuit only if both L and C are present .Only then do the voltages across L and C cancel each other.

We cannot have resonance in RL and RC circuit.

7.7 Power In AC Circuit: The Power Factor

$$p = v\,i$$

$$p = v_m \sin \omega t \; i_m \sin(\omega t + \phi)$$

$$P = \frac{v_m i_m}{2}\left(\cos\phi - \cos(2\omega t + \phi)\right)$$

$$P = \frac{v_m i_m}{2}\cos\phi$$

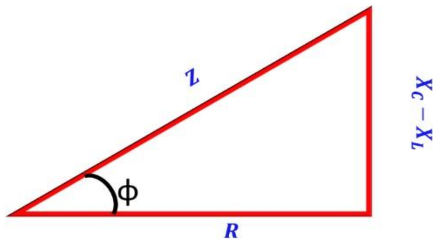
$$P = \frac{v_m}{\sqrt{2}}\frac{i_m}{\sqrt{2}}\cos\phi$$

$$P = V\,I\cos\phi$$

So, the average power dissipated depends not only on the voltage and current but also on the cosine of the phase angle ϕ between them. The quantity $\cos\phi$ is called the power factor.

Power factor can be obtained from impedance diagram.

R



cosϕ =
Z

Case (i)

Resistive circuit: $\phi = 0$,

$P = \mathbf{V}\,\mathbf{I}\cos 0 = VI$

There is maximum power dissipation.

Case (ii) Purely inductive or capacitive circuit:

$\phi = \pi/2$

$P = \mathbf{V}\,\mathbf{I}\cos \pi/2 = 0$

No power is dissipated even though a current is flowing in the circuit.

This current is sometimes referred to as wattless current.

Case (iii) LCR series circuit:

$$\phi = \tan^{-1}\frac{X_C - X_L}{R}$$

P= V I cos ϕ

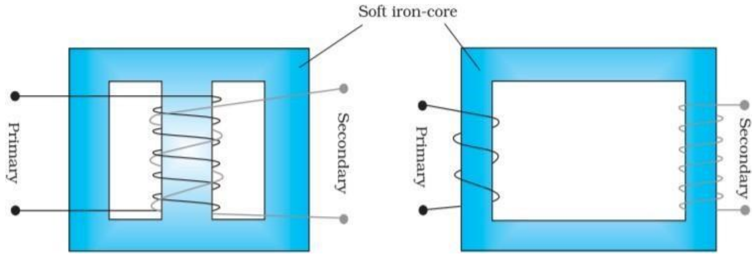
So, ϕ may be non-zero and power may dissipate in a RL or RC or RCL circuit. Even in such cases, power is dissipated only in the resistor.

Case (iv) Power dissipated at resonance in LCR circuit: At resonance $\mathbf{X_C - X_L} = 0$, and $\phi = 0$.

P= V I cos 0 =PV

That is, maximum power is dissipated in a circuit (through R) at resonance.

7.8 Transformer



A transformer consists of two sets of coils, insulated from each other. They are wound on a soft-iron core, One of the coils called the primary coil has N_P turns. The other coil is called the secondary coil; it has N_S turns. Often the primary coil is the input coil and the secondary coil is the output coil of the transformer.

Transformer works on the Principle of Mutual Induction

When an alternating voltage is applied to the primary, the resulting current produces an alternating magnetic flux which links the secondary and induces an emf in it.

The emf induced in the primary

$$\varepsilon_P = -N_P \frac{d\phi}{dt}$$

If the primary coil has negligible resistance $\varepsilon_P = V_P$(input voltage)

$$V_P = -N_P \frac{d\phi}{dt} \text{ ---------- } (1)$$

The emf induced in thesecondary

$$\varepsilon_s = -N_s \frac{d\phi}{dt}$$

If the secondary coil has negligible resistance $\varepsilon_s = V_s$(output voltage)

$$V_s = -N_s \frac{d\phi}{dt} \text{ ----------------- } (2)$$

$$\frac{\textbf{eq}\,(1)}{\textbf{eq}\,(2)} \text{ — — — } \quad \frac{V_s}{V_P} = \frac{N_s}{N_P} \text{ ------------------- } (3)$$

If the transformer is 100% efficient
power input = power output

$$I_P V_P = I_s V_s$$

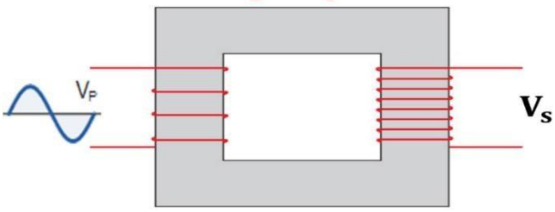
$$\frac{I_P}{I_s} = \frac{V_s}{V_P} \text{ -------------- } (4)$$

Combining equations (3)*and* (4)

$$\frac{I_P}{I_s} = \frac{V_s}{V_P} = \frac{N_s}{N_P}$$

Step-up Transformer

$N_s > N_P$



For a step up transformer the number of turns in the secondary will be greater than that in the primary($N_S > N_P$)

$$V_s = \left(\frac{N_s}{N_P}\right) V_p \qquad I_s = \left(\frac{N_P}{N_s}\right) V_p$$

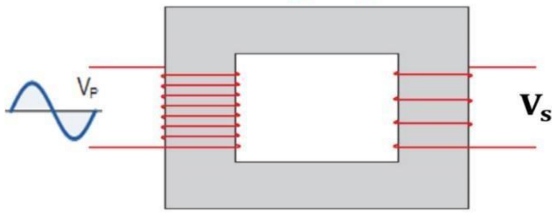
$$\left(\frac{N_s}{N_P}\right) > 1 \qquad \left(\frac{N_P}{N_s}\right) < 1$$

$$V_s > V_p \qquad I_s < I_p$$

Thus for a step up transformer secondary voltage will be greater than primary voltage,but the secondary current will be less than primary current.

Step-down Transformer

$N_s < N_P$



For a step down transformer the number of turns in the secondary will be less than that in the primary($N_S < N_P$)

$$V_s = \left(\frac{N_s}{N_P}\right) V_p \qquad I_s = \left(\frac{N_P}{N_s}\right) V_p$$

$$\left(\frac{N_s}{N_P}\right) < 1 \qquad \left(\frac{N_P}{N_s}\right) > 1$$

$$V_s < V_p \qquad I_s > I_p$$

Thus for a step up transformer secondary voltage will be less than primary voltage, but the secondary current will be greater than primary current.

Energy Losses in a Transformer

(i) Flux Leakage:

There is always some flux leakage; that is, not all of the flux due to primary passes through the secondary due to poor design of the core or the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

(ii) Resistance of the windings :

The wire used for the windings has some resistance and so, energy is lost due to heat produced in the wire(I^2R). In high current, low voltage windings, these are minimised by using thick wire.

(iii) Eddy currents loss:

The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by having a laminated core.

(iv) Hysteresis loss:

The magnetisation of the core is repeatedly reversed by the alternating
magnetic field. This produces hysteresis and energy is lost as heat. This can be minimised by using a magnetic material which has a low hysteresis loss(e.g- soft iron core)