Chapter 10
Wave Optics

10.1 Introduction

In 1678, the Dutch physicist Christiaan Huygens put forward the wave theory of light .The wave model could satisfactorily explain the phenomena of reflection , refraction, interference, diffraction and polarisation .

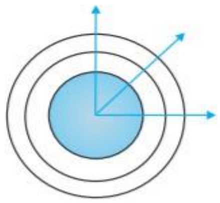
Wavefront

A locus of points, which oscillate in phase is called a wavefront; thus a wavefront is defined as a surface of constant phase.

The speed with which the wavefront moves outwards from the source is called the speed of the wave. The energy of the wave travels in a direction perpendicular to the wavefront.

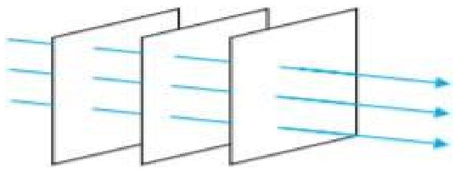
Spherical Wavefront

For a point source emitting waves uniformly in all directions, the wavefronts will be spherical .



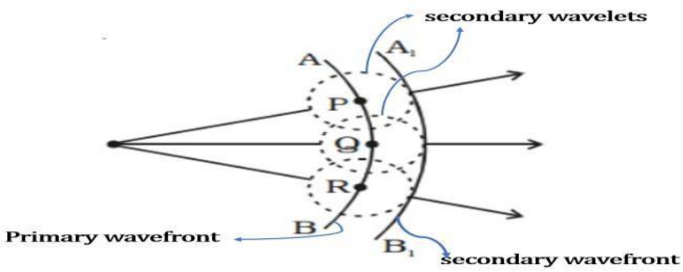
Plane Wavefront

At large distance from a source, a small portion of the sphere can be considered as a plane and is known as a plane wavefront.



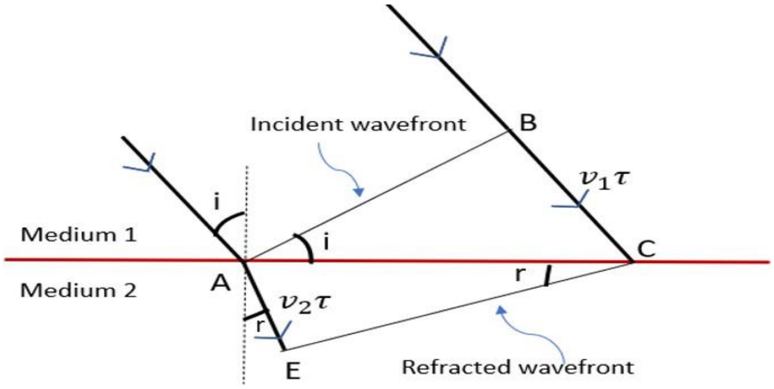
10.2 Huygens Principle

According to Huygens principle, each point of the wavefront acts as a source secondary wavelets and if we draw a common tangent to all these secondary wavelets, we obtain the new position of the wavefront at a later time.



10.3 Refraction and Reflection of Plane Waves Using Huygens Principle

Refraction of a Plane Wave



AB is the incident wavefront and EC is the refracted wavefront. Let v_1 and v_2 be the velocity of wave in medium 1 and 2 respectively.

$$\sin i = \frac{BC}{AC} = \frac{v_1\tau}{AC} \text{ --------------- } (1)$$

$$\sin r = \frac{AE}{AC} = \frac{v_2\tau}{AC} \text{ --------------- } (2)$$

$$\text{eqn}\frac{(1)}{(2)} \qquad \frac{\sin i}{\sin r} = \frac{v_1}{v_2} \text{ ---------------------- } (3)$$

Refractive index of first medium $\quad n_1 = \frac{c}{v_1}$

Refractive index of second medium $\quad n_2 = \frac{c}{v_2}$

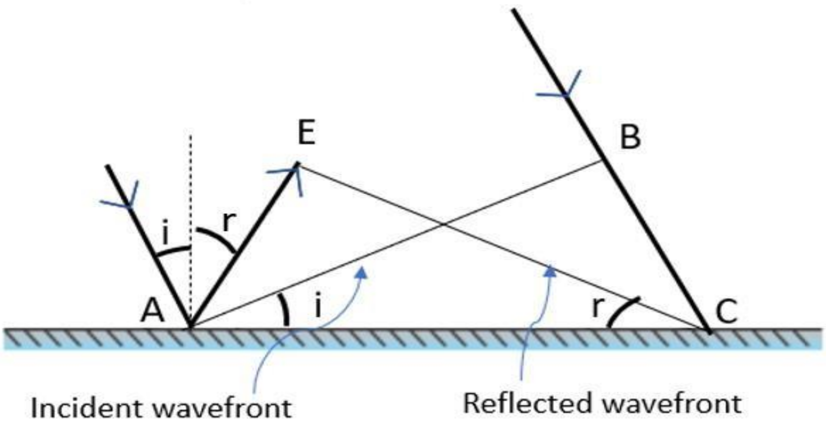
$$\frac{n_2}{n_1} = \frac{v_1}{v_2}$$

Substituting in eqn (3)

$$\mathbf{\frac{\sin i}{\sin r} = \frac{n_2}{n_1}}$$

This is the Snell's law of refraction.

Reflection of a plane wave by a plane surface



AB is the incident wavefront and EC is the reflected wavefront.

Let v be the velocity of the wave ,then

$$AE = BC = v\tau$$

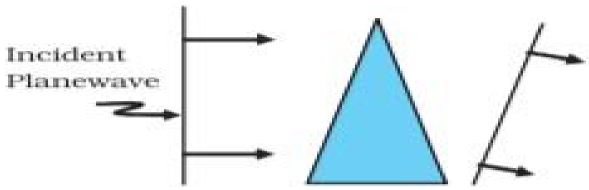
$$AC = AC \ \text{(common side)}$$

So the triangles EAC and BAC are congruent . Therefore

$$i = r$$

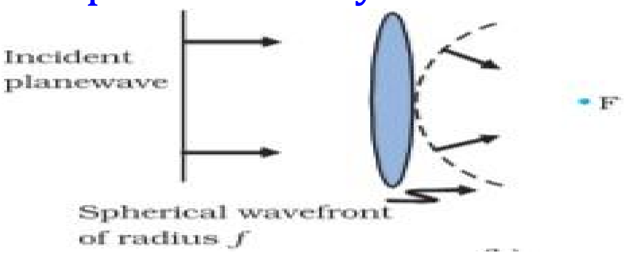
Angle of incidence=Angle of reflection
This is the law of reflection.

Refraction of a plane wave by a thin prism



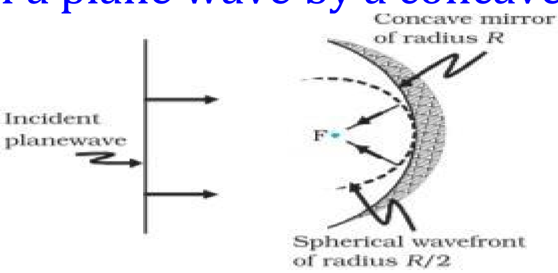
The emerging wavefront is also plane wavefront,but tilted.

Refraction of a plane wave by a convex lens



The emerging wavefront is spherical and converges to the point F which is known as the focus.

Reflection of a plane wave by a concave mirror



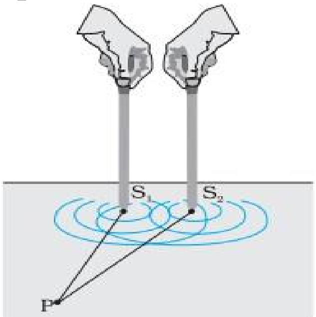
The reflected wavefront is a spherical converging to the focal point F.

Superposition Principle

According to superposition principle , the resultant displacement produced by a number of waves in a medium is the vector sum of the displacements produced by each of the waves.

10.4 Coherent sources

Two sources are said to be coherent if they emit light waves of same frequency and same phase or constant phase difference.



Two needles oscillating in phase in water represent two coherent sources.

Interference

Interference is the phenomenon in which two waves superpose to form a resultant wave of greater or lower amplitude.

The interference can be constructive or destructive.

Coherent and Incoherent Addition of Waves

Coherent Addition of Waves

Consider two coherent sources S_1 and S_2. The displacement produced by the source S_1 and S_2 at the point P has a phase difference ϕ.

$$y_1 = a \cos \omega t$$

$$y_2 = a \cos (\omega t + \phi)$$

The resultant displacement

$$y = y_1 + y_2 = a [\cos \omega t + \cos (\omega t + \phi)]$$

$$= 2 a \cos (\phi /2) \cos (\omega t + \phi/2)$$

The amplitude of the resultant displacement is $2a \cos (\phi/2)$

$$\text{Intensity} \propto (\text{amplitude})^2$$

$$I \propto (2a \cos (\phi /2))^2$$

$$I \propto 4a^2 \cos^2 (\phi /2)$$

$$I \propto 4I_0 \cos^2 (\phi /2)$$

If $\phi = 0, \pm 2\pi, \pm 4\pi, \ldots$

We will have constructive interference leading to maximum intensity.

If $\phi = \pm \pi, \pm 3\pi, \pm 5\pi \ldots$

We will have destructive interference leading to zero intensity.

As the two sources are coherent, the phase difference ϕ at any point will not change with time and we will have a stable interference pattern; i.e., the positions of maxima and minima will not change with time.

Incoherent Addition of Waves

If the two sources are incoherent then the phase difference ϕ between the two vibrating sources changes rapidly with time .The positions of maxima and minima will also vary rapidly with time and we will see a "time-averaged" intensity distribution.

$$\langle I \rangle = 4I_0 \langle \cos^2 (\phi /2) \rangle$$

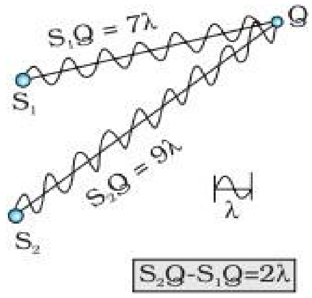
$$= 4I_0 \times \frac{1}{2}$$

$$= 2I_0$$

i.e. ,the intensities just add up.

This is indeed what happens when two separate light sources illuminate a wall.

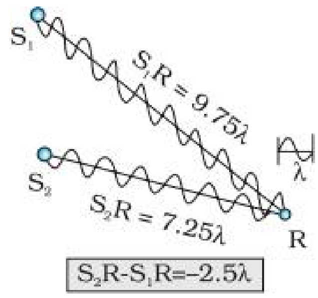
Condition for constructive interference



If the path difference at a point is an integral multiple of λ, there will be constructive interference and a bright fringe is formed at that point

$$S_2P - S_1P = n\lambda \qquad \text{where } (n = 0, 1, 2, 3,...)$$

Condition for destructive interference



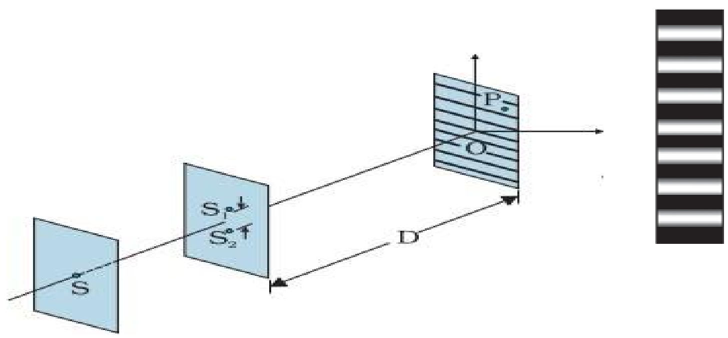
If the path difference at a point is an odd integral multiple of λ/2 , there will be destructive interference and a dark fringe is formed at that point

$$S_2P - S_1P = = (n+ \frac{1}{2})\,\lambda \qquad \text{where } (n = 0, 1, 2, 3,...)$$

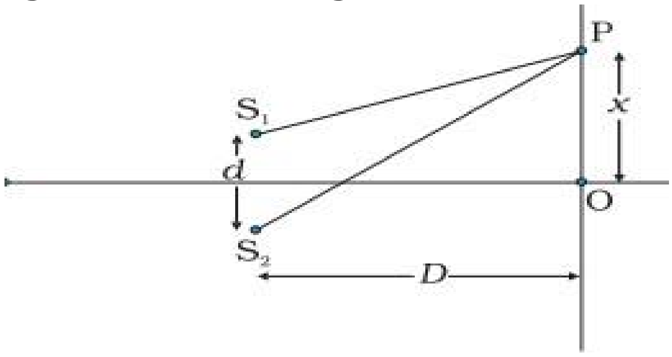
Two sodium lamps illuminating two pinholes cannot produce interference fringes. Why?

If we use two sodium lamps illuminating two pinholes we will not observe any interference fringes. This is because the light wave emitted from an ordinary source (like a sodium lamp) undergoes abrupt phase changes .Thus the light waves coming out from two independent sources of light will not have any fixed phase relationship and would be incoherent and cannot produce interference pattern.

10.5 Interference of Light Waves and Young's Experiment



The British physicist Thomas Young made two pinholes S_1 and S_2 (very close to each other) on an opaque screen. These were illuminated by another pinholes which is illuminated by a bright source. Light waves spread out from S and fall on both S_1 and S_2. S_1 and S_2 then behave like two coherent sources because light waves coming out from S_1 and S_2 are derived from the same original source and interference pattern with altermate bright and dark fringes is formed on the screen.

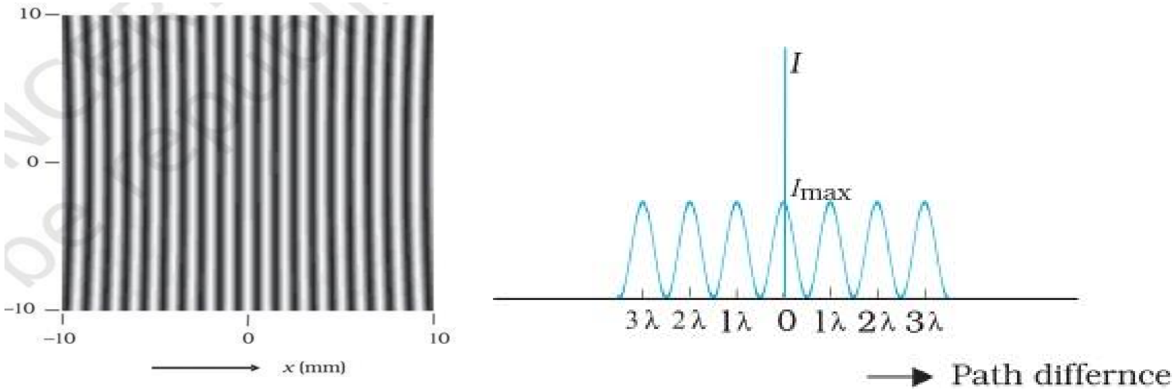


For bright band path difference , $\frac{xd}{D} = n\lambda$

$x_n = \frac{n\lambda D}{d}$, $n = 0, \pm 1, \pm 2, ...$

For dark band path difference, $\frac{xd}{D} = (n + \frac{1}{2})\lambda$

$x_n = (n + \frac{1}{2})\frac{\lambda D}{d}$, $n = 0, \pm 1, \pm 2, ...$



Dark and bright bands appear on the screen are called fringes. Dark and bright fringes are equally spaced.

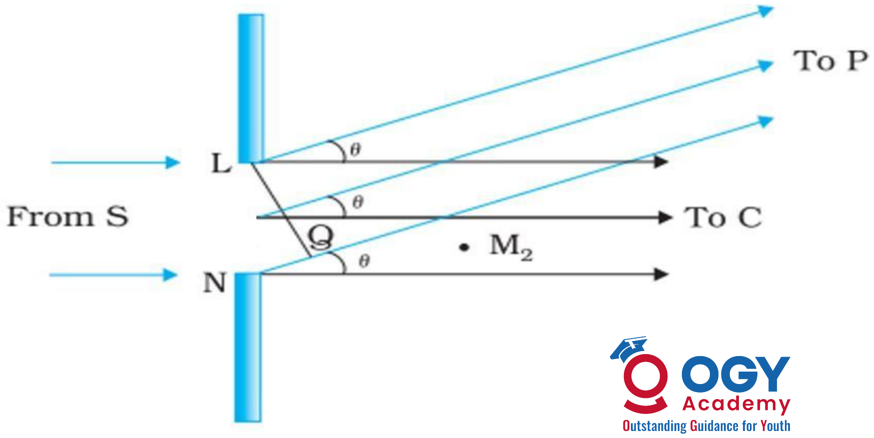
10.6 Diffraction

Diffraction is the phenomenon of bending of light around the corners of an obstacle , into the region of geometrical shadow of the obstacle.

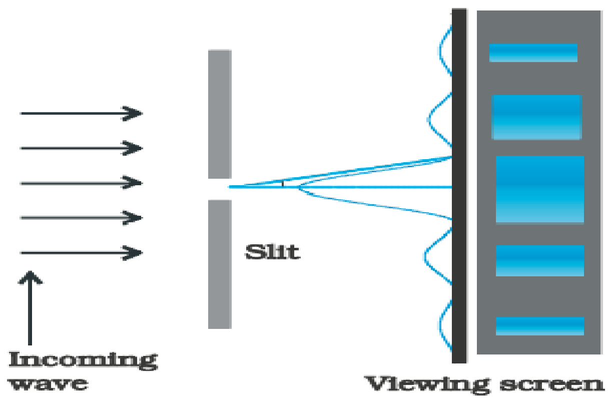
If we look clearly at the shadow cast by an opaque object, close to the region of geometrical shadow, there are alternate dark and bright regions just like in interference. This happens due to the phenomenon of diffraction. Diffraction is a general characteristic exhibited by all types of waves, be it sound waves, light waves, water waves or matter waves.

- Since the wavelength of light is much smaller than the dimensions of most obstacles; we do not encounter diffraction effects of light in everyday observations.
- The resolving power of our eye ,telescopes and microscopes are limited due to diffraction.
- The colours seen on CD are due to diffraction.

The single slit



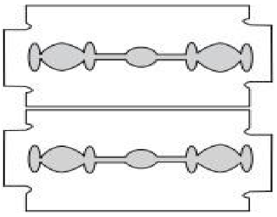
When a single narrow slit is illuminated by a monochromatic source, a broad pattern with a central bright region (central maximum) is seen on the screen. On both sides, there are alternate dark and bright regions(secondary maxima and secondary minima), the intensity becoming weaker away from the centre. This is diffraction pattern.



For central maximum, angle $\theta = 0$

For secondary maxima, $\theta = (n + \frac{1}{2})\frac{\lambda}{a}$ where $n = \pm 1, \pm 2, \pm 3, ...$

For secondary minima, $\theta = n\frac{\lambda}{a}$ where $n = \pm 1, \pm 2, \pm 3, ...$



Holding two blades to form a single slit. A bulb filament viewed through this shows clear diffraction bands.

Cosistency with Principle of Conservation of Energy.

In interference and diffraction, light energy is redistributed. If it reduces in one region, producing a dark fringe, it increases in another region, producing a bright fringe. There is no gain or loss of energy, which is consistent with the principle of conservation of energy.

10.7 Polarisation

A wave propagating in x direction in a horizontally string ,with displacement in y direction can be represented as

$$y(x,t) = a \sin(kx - \omega t)$$

It is referred to as a y-polarised wave.

Since each point on the string moves on a straight line, the wave is also referred to as a linearly polarised wave.

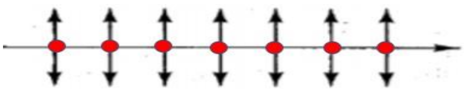
As the string always remains confined to the x-y plane , it is also referred to as a plane polarised wave.

Polarisation of Light

The phenomenon of restricting the electric field vibrations of light to one plane is called polarisation.

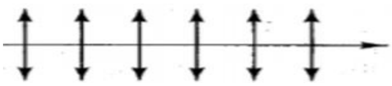
Unpolarised Light

For an unpolarised light the vibrations of electric vector takes all possible directions in the transverse plane. Natural light, e.g., from the sun is unpolarised.



Plane Polarised Light

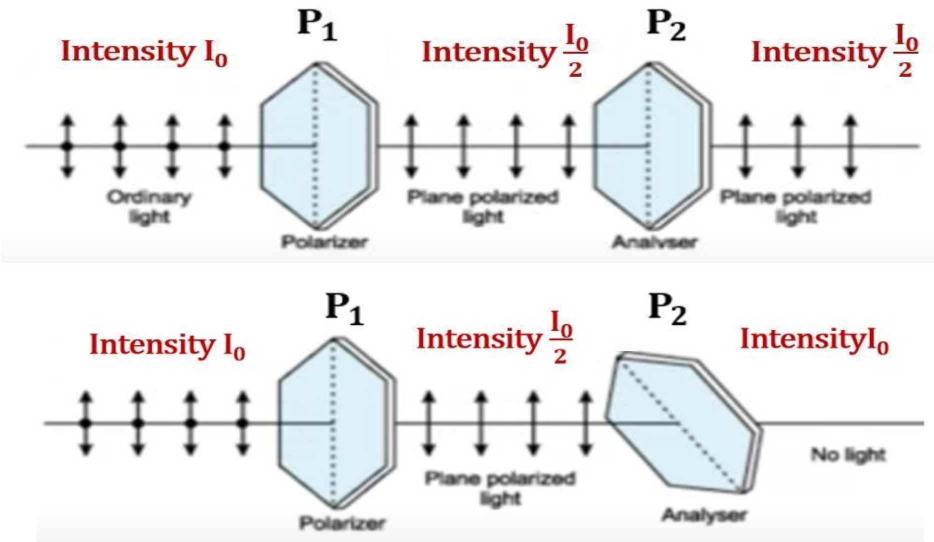
For a plane polarised light the vibrations of electric field vector are restricted in one direction .



Polaroids

Polaroids are thin plastic like sheets, which consists of long chain molecules aligned in a particular direction. The electric vectors along the direction of the aligned molecules get absorbed. Thus, if an unpolarised light wave is incident on a polaroid ,it transmits only one component of electric field vectors which are parallel to its pass axis and the resulting light is c linearly polarised or plane polarised.

Polaroids are used in sunglasses, wind screens in trains and aeroplanes, in 3D cameras.



Malus' Law

When an unpolarised light is passed through two polaroids P_1 and P_2 and if the angle between the polaroids is varied from 0º to 90º, the intensity of the transmitted light will vary as:

$$I = I_0 \cos^2\theta$$

where I_0 is the intensity of the polarized light after passing through P_1. This is known as Malus' law.