Chapter 13
Nuclei

13.1 Introduction

The volume of a nucleus is about 10^{-12} times smaller than the volume of the atom. In other words, an atom is almost empty. If an atom is enlarged to the size of a classroom, the nucleus would be of the size of pinhead. Nevertheless, the nucleus contains most (more than 99.9%) of the mass of an atom.

13.2 Atomic Masses and Composition of Nucleus

The mass of an atom is very small. Kilogram is not a very convenient unit to measure such small quantities. Therefore, a different mass unit is used for expressing atomic masses. This unit is the atomic mass unit (u).

Atomic Mass Unit (u)

Atomic mass unit (u) is defined as 1/12th of the mass of the carbon (^{12}C) atom.

$$1u = \frac{\text{mass of the one C}-12\text{ atom}}{12}$$

$$= \frac{1.992647 \, x \, 10^{-26}}{12}$$

$$= 1.660539 \, 10^{-27} \text{ kg}$$

Accurate measurement of atomic masses is carried out with a mass spectrometer.

Composition of Nucleus

The composition of a nucleus can be described using the following terms and symbols:

Z - atomic number = number of protons = number of electrons
N - neutron number = number of neutrons=A-Z
A - mass number = Z + N = total number of protons and neutrons .

Protons and neutrons are also called nucleons. Thus the number of nucleons in an atom is its mass number A.

- All the electrons of an atom are outside the nucleus.
- The total charge of the atomic electrons is (–Ze)
- The total cherge of the nucleus is (+Ze).
- Atom is electrically neutral
- The mass of proton , m_p =1.00727 u = 1.67262 10^{-27} kg
- The mass of neutron , m_n = 1.00866 u = 1.6749×10^{-27} kg

- Neutron was discovered by James Chadwick.
- Chadwick was awarded the Nobel Prize in Physics in 1935 for his discovery of the neutron.

Isotopes

Isotpes are different types of atoms of the same element, with same atomic number ,but different mass number .They exhibit the same chemical properties, but differ in mass.

- Chlorine has two isotopes having masses 34.98 u and 36.98 u. The relative abundances of these isotopes are 75.4 and 24.6 per cent, respectively. Thus, the average mass of a chlorine atom is obtained by the weighted average of the masses of the two isotopes, which works out to be

$$= \frac{75.4 \times 34.98 + 24.6 \times 36.98}{\mathbf{100}}$$

$$= 35.47 \text{ u}$$

which agrees with the atomic mass of chlorine.

- The lightest element, hydrogen has three isotopes ,
  - Proton(^1_1H) - contains one proton only
  - Deuterium(^2_1H) - contains one proton and one neutron.
  - Tritium(^3_1H) - Contains one proton and two neutrons.

  Tritium nuclei, being unstable, do not occur naturally and are produced artficially in laboratories.

- The element gold has 32 isotopes, ranging from
  A =173 to A = 204.

Isobars

All nuclides with same mass number A , but with different atomic number are called isobars.

For example, the nuclides (^3_1H) and (^3_2He)are isobars.

Isotones

Nuclides with same neutron number N but different atomic number Z are called isotones.

For example $^{198}_{80}Hg$ $^{197}_{79}Au$ are isotones.

13.3 Size of The Nucleus

By performing scattering experiments in which fast electrons, instead of α-particles, are projectiles that bombard targets made up of various elements, the sizes of nuclei of various elements have been accurately measured.

Radius of nucleus

A nucleus of mass number A has a radius

$$R = R_0 A^{1/3} \qquad \text{where } R_0 = 1.2 \times 10^{-15}\text{ m.}$$

Volume of nucleus

$$V = \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi \left(\mathbf{R_0 A^{1/3}}\right)^3$$

$$= \frac{4}{3}\pi(\mathbf{R_0})^3\mathbf{A}$$

The volume of the nucleus is proportional to A

Density of nucleus

$$\text{Nuclear density} = \frac{\text{mass}}{\text{volume}}$$

$$= \frac{A m_p}{\frac{4}{3}\pi(\mathbf{R_0})^3\mathbf{A}}$$

$$= \frac{m_p}{\frac{4}{3}\pi(\mathbf{R_0})^3} = \text{constant}$$

Thus the density of nucleus is a constant, independent of A, for all nuclei.

Example

Given the mass of iron nucleus as 55.85u and A=56, find the nuclear density?

$$m_{Fe} = 55.85\text{ u} = 9.27 \times 10^{-26}\text{ kg}$$

$$\text{Nuclear density} = \frac{m_{Fe}}{\frac{4}{3}\pi(\mathbf{R_0})^3 \boldsymbol{A}}$$

$$= \frac{9.27 \times 10{-}26}{\frac{4}{3}\pi(1.2 \times 10{-}15\,)^3 \times 56}$$

$$= 2.29 \times 10^{17}\text{ kg m}^{-3}$$

13.4 Mass- Energy and Nuclear Binding Energy

Mass – Energy

Einstein showed that mass is another form of energy and one can convert mass-energy into other forms of energy, say kinetic energy and vice-versa.

Einstein gave the famous mass-energy equivalence relation

$$E = mc^2$$

c is the velocity of light in vacuum.c= 3×10^8 m s^{-1}.

Experimental verification of the Einstein's mass-energy relation has been achieved in the study of nuclear reactions. In a reaction the conservation law of energy states that the initial energy and the final energy are equal provided the energy associated with mass is also included.

Example

Calculate the energy equivalent of 1 g of substance.

$$E = mc^2$$
$$= 1x10^{-3} \times (3 \times 10^8)^2$$
$$= 10^{-3} \times 9 \times 10^{16}$$
$$= 9 \times 10^{13}\ J$$

Thus, if one gram of matter is converted to energy, there is a release of enormous amount of energy.

Example

Find the energy equivalent of one atomic mass unit, first in Joules and then in MeV.

$$1u = 1.6605 \times 10^{-27}\ kg$$
$$E = mc^2$$
$$= 1.6605 \times 10^{-27}\ x (3\ x\ 10^8)^2$$
$$E = 1.4924 \times 10^{-10}\ J$$

Energy equivalent in MeV.

$$1eV = 1.602\ x10^{-19} J$$
$$E = \frac{1.4924 \times 10-10}{1.602\ x10-19}$$
$$= 0.9315 \times 109\ eV$$
$$= 931.5\ MeV$$

Mass Defect and Binding Energy

Mass Defect (ΔM)

The nucleus is made up of neutrons and protons. Therefore it may be expected that the mass of the nucleus is equal to the total mass of its individual protons and neutrons.

The nuclear mass M is always less than the total mass, of its constituents (protons and neutrons). The difference in mass of a nucleus and its constituents is called the mass defect.

$$\Delta M = [\mathbf{Z\ m_p + (A - Z) m_n}] - \mathbf{M}$$

For example, let us consider ${}^{16}_{8}O$; a nucleus which has 8 neutrons and 8 protons.

Mass of 8 neutrons = 8 × 1.00866 u
Mass of 8 protons = 8 × 1.00727 u
The expected mass of $^{16}_{8}O$ nucleus = 16.12744 u

The atomic mass of $^{16}_{8}O$ from mass spectroscopy experiments= 15.99493 u
Substracting the mass of 8 electrons (8 × 0.00055 u)
The mass of $^{16}_{8}O$ nucleus =15.99493 u -(8 × 0.00055 u)= 15.99053 u

Mass defect ΔM =16.12744 -15.99053 u
= 0.13691u

Binding Energy

The energy equivalent of mass defect is called binding energy.

$E_b = \Delta Mc^2$

- If we separate a nucleus into its nucleons, we would have to supply a total energy equal to E_b, to those particles.
- If a certain number of neutrons and protons are brought together to form a nucleus of a certain charge and mass, an energy E_b will be released .

Binding Energy Per Nucleon

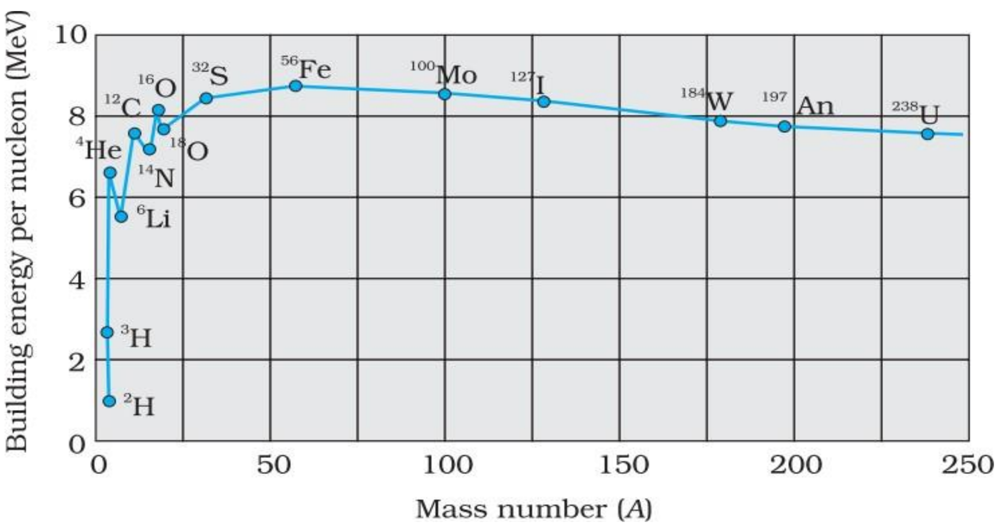
A more useful measure of the binding between the constituents of the nucleus is the binding energy per nucleon, E_{bn},

The binding energy per nucleon, E_{bn}, is the ratio of the binding energy Eb of a nucleus to the number of the nucleons, A, in that nucleus.

$$E_{bn} = E_b / A$$

It is the average energy per nucleon needed to separate a nucleus into its individual nucleons.

The plot of binding energy per nucleon versus mass number



Observations:

(i) In the mass number range A = 30 to 170 ($30 < A < 170$), the binding energy per nucleon is nearly constant, about 8 MeV/nucleon.

(ii) The maximum of about 8.75 MeV for A = 56 i,e,.for ^{56}Fe nucleus.

(iii) E_{bn} is lower for both light nuclei with $A< 30$ and for heavy nuclei with $A>170$.

We can draw some conclusions from these two observations:

(i) The nuclear force is attractive and sufficiently strong to produce a binding energy of a few MeV per nucleon.

(ii) The constancy of the binding energy in the range $30 < A < 170$ indicates that the nuclear force is short-ranged. A nucleon influences only nucleons close to it and this property is called saturation property of the nuclear force.

(iii) A very heavy nucleus, say A = 240, has lower binding energy per nucleon .Such a heavy nucleus breaks into two lighter nuclei, thereby increasing the binding energy per nucleon and the nucleons get more tightly bound. Energy would be released in the process and this is an implication of through fission.

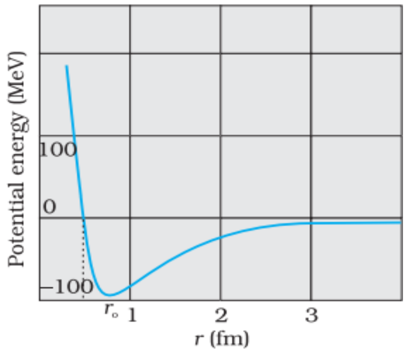
(iv) Two very light nuclei ($A \leq 10$) have lower binding energy per nucleon .They join to form a heavier nucleus , thereby increasing the binding energy per nucleon and the nucleons get more tightly bound. Energy would be released in such a process and this is an implication of through of fusion.

13.5 Nuclear Force

The nuclear force binds the nucleons together inside the nucleus.

(i) The nuclear force is much stronger than the Coulomb repulsive force between protons inside the nucleus and the gravitational force between the masses.

(ii) The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few femtometres. The force is attractive for distances larger than 0.8 fm and repulsive if they are separated by distances less than 0.8 fm.



A rough plot of the potential energy between two nucleons as a function of distance. The potential energy is a minimum at a distance r_0 of about 0.8 fm.

(iii) The nuclear force between neutron-neutron, proton-neutron and proton-proton is approximately the same. The nuclear force does not depend on the electric charge.

13.6 Radioactivity

A .H. Becquerel discovered radioactivity in 1896. Radioactivity is a nuclear phenomenon in which an unstable nucleus undergoes a decay.This is referred to as radioactive decay.

Three types of radioactive decay occur in nature :

α-decay in which a helium nucleus (He) is emitted;
β-decay in which electrons or positrons (particles with the same mass as electrons, but with a charge exactly opposite to that of electron) are emitted;
γ-decay in which high energy (hundreds of keV or more) photons are emitted.

13.7 Nuclear Energy

Energy then can be released if less tightly bound nuclei are transmuted into more tightly bound nuclei. Two such processes, are fission and fusion.

For the same quantity of matter, nuclear sources will give a million times larger energy than conventional sources. One kilogram of coal on burning gives 10^7 J of energy, whereas 1 kg of uranium, which undergoes fission, will generate on fission 10^{14} J of energy.

Nuclear Fission

Nuclear fission is the process in which a heavier nucleus splits into lighter nuclei with the release of large amount of energy.

When a neutron was bombarded on a uranium target, the uranium nucleus broke into two nearly equal fragments releasing great amount of energy.

Example

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3\,{}^{1}_{0}n $$

Fission does not always produce barium and krypton. A different pair can be produced,

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{133}_{51}Sb + {}^{99}_{41}Nb + 4\,{}^{1}_{0}n $$

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\,{}^{1}_{0}n $$

The energy released (the Q value) in the fission reaction of nuclei like uranium is of the order of 200 MeV per fissioning nucleus.

The enormous energy released in an atom bomb comes from uncontrolled nuclear fission.

Nuclear Fusion – Energy Generation in Stars

Nuclear fusion is the process in which two light nuclei combine to form a single larger nucleus, with the release of a large amount of energy.

Examples are

$$ {}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + e^{+} + \nu + 0.42\ MeV $$

$$ {}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{2}_{3}He + n + 3.27\ MeV $$

$$ {}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H + 4.03\ MeV $$

Thermonuclear fusion

For nuclear fusion to occur in bulk matter the temperature of the material is to be raised until the particles have enough energy to penetrate the coulomb barrier. This process is called thermonuclear fusion.

for thermonuclear fusion to take place, extreme conditions of temperature and pressure are required, which are available only in the interiors of stars including sun.

The energy generation in stars takes place via thermonuclear fusion.

The fusion reaction in the sun is a multi-step process called the proton-proton (p, p) cycle.

$$ {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} \rightarrow {}^{2}_{1}\text{H} + e^{+} + \nu + 0.42\ \text{MeV} $$

$$ e^{+} + e^{-} \rightarrow \gamma + \gamma + 1.02\ \text{MeV} $$

$$ {}^{2}_{1}\text{H} + {}^{1}_{1}\text{H} \rightarrow {}^{2}_{3}\text{He} + \gamma + 5.49\ \text{MeV} $$

$$ {}^{3}_{2}\text{H} + {}^{3}_{2}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} + 12.86\ \text{MeV} $$

The combined reaction is

$$ 4\ {}^{1}_{1}\text{H} + 2\ e^{-} \rightarrow {}^{4}_{2}\text{He} + 2\nu + 6\gamma + 26.7\ \text{MeV} $$

$$ \text{Or } (4\ {}^{1}_{1}\text{H} + 4\ e^{-}) \rightarrow ({}^{4}_{2}\text{He} + 2e^{-}) + 2\nu + 6\gamma + 26.7\ \text{MeV} $$

Thus, four hydrogen atoms combine to form an 4 2He atom with a release of 26.7 MeV of energy.

In about 5 billion years, however, the sun's core, which by that time will be largely helium, will begin to cool and the sun will start to collapse under its own gravity. This will raise the core temperature and cause the outer envelope to expand, turning the sun into what is called a red giant.