Chapter 3

Current Electricity

3.1 Introduction

Charges in motion constitute an electric current. Such currents occur naturally in many situations. Lightning is one such phenomenon in which charges flow from the clouds to the earth through the atmosphere. The flow of charges in lightning is not steady, but in our everyday life we see many devices where charges flow in a steady manner. A torch and a cell- driven clock are examples of such devices.

3.2 Electric Current

When current steady ,
The rate of flow of charge through any cross-section of a conductor is called electric current flowing through it.

$$I = \frac{q}{t}$$

Unit of electric current $= \frac{\text{coulomb}}{\text{second}} = \text{C/s} = \text{ampere (A)}$

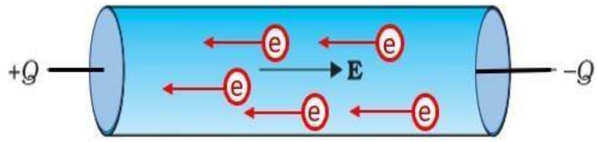
When current is not steady,
The current at time t across the cross-section of the conductor is defined as the ratio of ΔQ to Δt in the limit of Δt tending to zero, $I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

3.3 Electric Currents in Conductors

When no electric field is present:-

The electrons will be moving due to thermal motion . During motion electrons collide with the fixed ions. The direction of its velocity after the collision is completely random. The average velocity of electrons will be zero. So, there will be no net electric current.

When an electric field is present:-



The electrons will be accelerated due to this field towards +Q. They will thus move to neutralise the charges and constitute an electric current. Hence there will be a current for a very short while and no current thereafter.

To maintain a steady electric field in the body of the conductor we use cells or batteries.

3.4 Ohm's Law

A basic law regarding flow of currents was discovered by G.S. Ohm in 1828.
At constant temperature ,the current flowing through a conductor is directly proportional to the potential difference between the ends of the conductor.

$$V \propto I$$

$$V = RI$$

$$\mathbf{R = \frac{V}{I}}$$

The constant of proportionality R is called the resistance of the conductor
The SI units of resistance is ohm and is denoted by the symbol Ω.

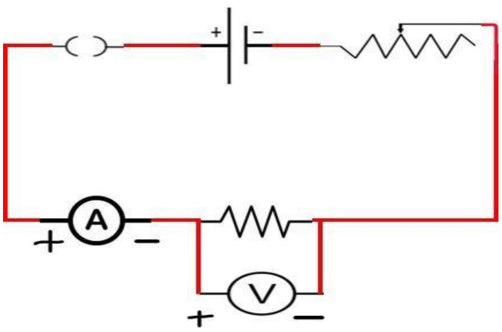
Conductance

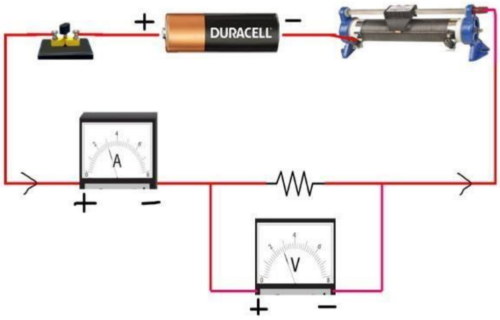
The reciprocal of resistance is called Conductance.

$$\mathbf{C = \frac{I}{V}}$$

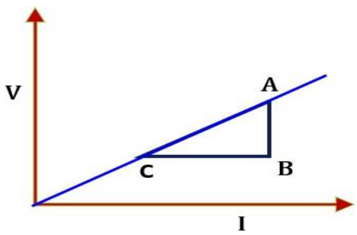
Unit of conductance is ohm^{-1}(Ω^{-1}or mho) or =siemens Ohm's

Law : Experimental verification





Voltage –Current Graph (V-I Graph)

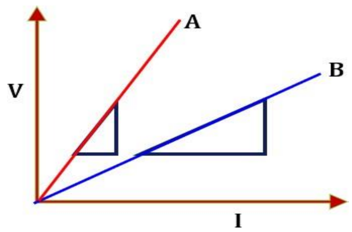


$$Slope = \frac{AB}{BC}$$

$$Slope = \frac{V}{I} = R$$

Slope of V-I graph gives Resistance.
Its reciprocal gives conductance.

Which material has more resistance?



Slope of V-I graph gives Resistance. Slope of A is greater than that of B.
So material A has more resistance than B.

Factors on which the Resistance of a Conductor Depends:-

1) The material of the conductor 2) The dimensions of the conductor a) Length of the conductor

The resistance of a conductor is directly proportional to length l of the conductor.

R a ***l***

b) The area of cross section of the conductor

The resistance of a conductor is inversely proportional to the cross-sectional area, A.

R a $\frac{1}{A}$

Resistivity of a Conductor

The resistance of a conductor is directly proportional to length l of the conductor and inversely proportional to the cross-sectional area, A.

$$R \propto \frac{l}{A}$$

$$\mathbf{R} = \frac{\rho l}{A}$$

where the constant of proportionality ρ is called resistivity.

Resistivity depends on the material of the conductor but not on its dimensions.

$$\rho = \frac{RA}{l}$$

$$\text{Unit of resistivity} = \frac{\Omega m^2}{m} = \Omega m$$

Ohm's Law in Vector Form

$V = IR$

$R = \frac{\rho l}{A}$

$V = \frac{I\rho l}{A}$

$V = j\rho l$

Current density

Current per unit area (taken normal to the current), is called current density and is denoted by j.

Current density $j = \frac{I}{A}$

Unit of current density $= A/m^2$

Current density is a vector quantity.

If E is the magnitude of uniform electric field in the conductor whose length is l, then the potential difference V across its ends , $V = E\,l$.

$E\,l = j\rho l$

$\vec{E} = \vec{j}\rho$

This is the vector form of Ohm's law. Here electric field and current density are vector quantities

$\vec{j} = \frac{\vec{E}}{\rho}$

$\vec{j} = \sigma\vec{E}$

This is another equation for Ohm's law in vector form.

Conductivity

Conductivity is the reciprocal of resistivity

$\sigma = \frac{1}{\rho}$

Unit of conductivity is $\Omega^{-1}\, m^{-1}$

3.5 Drift of Electrons and the Origin of Resistivity

Drift Velocity

The average velocity attained by electrons in a conductor due to an electric field is called Drift velocity.

The force acting on the electron due to the electric field, $F = qE = -eE$

The acceleration of the electron,

$$a = \frac{F}{m} = \frac{-eE}{m}$$

If t is the time between two successive collissions, $v = at$

Then the velocity gained by an electron, $v = \frac{-eEt}{m}$

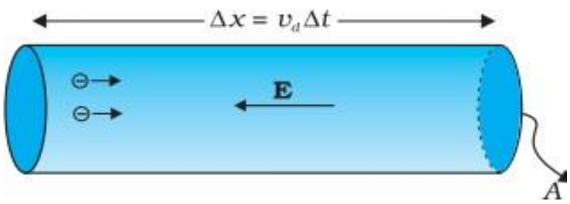
Drift velocity , $v_d = -\frac{eE}{m}(t)_{average}$

$$\mathbf{v_d = -\frac{eE}{m}\tau}$$

Relaxation Time

The average time interval between two successive collisions is called relaxation time(τ) $\mathbf{v_d}$ a $\boldsymbol{\tau}$

Relation connecting Drift Velocity and Current



Distance Travelled by an electron in time $\Delta t = v_d \Delta t$

Volume of conductor $= A\, v_d\, \Delta t$

Let n be the number of electrons per unit volume of conductor

The number of electrons in the conductor$= n\, A\, v_d\, \Delta t$

Total charge of electrons in the conductor , $q = n\, e\, A\, v_d\, \Delta t$

$$\text{Current } I = \frac{q}{\Delta t}$$

$I = \frac{n\,e\,A\,v_d\,\Delta t}{\Delta t}$

$I = n\,e\,A\,v_d$

Current density

$I = n\,e\,A\,v_d$

$v_d = \frac{eE}{m}\tau$

$I = n\,e\,A\,\frac{eE}{m}\tau$

$I = \frac{ne^2A\tau E}{m}$

$j = \frac{I}{A}$ $\quad j = \frac{ne^2\tau E}{m}$

Conductivity

Comparing with, $j = \sigma E$

$\sigma = \frac{ne^2\tau}{m}$

Mobility

Conductivity arises from mobile charge carriers.
In metals, these mobile charge carriers are electrons.
In an ionised gas, they are electrons and positive charged ions.
In an electrolyte, these can be both positive and negative ions.
Mobility μ defined as the magnitude of the drift velocity per unit electric field.

$\mu = \frac{v_d}{E} = \frac{\frac{eE}{m}\tau}{E}$

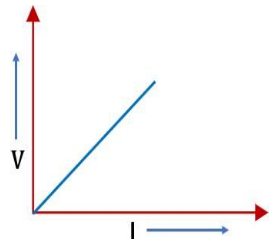
$\mu = \frac{e\tau}{m}$

3.6 Limitations of Ohm's Law

Ohmic Conductors

Conductors which obey Ohm's law are called Ohmic conductors.The Voltage – Current graph of such conductors will be linear .

Eg:- metals ,Nichrome



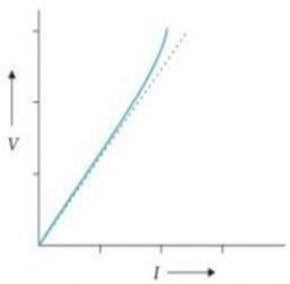
Non - Ohmic Conductors

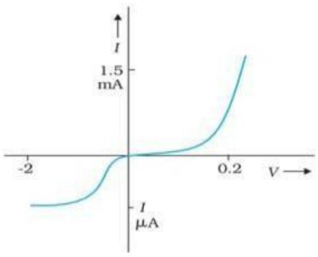
The materials and devices used in electric circuits which do not obey Ohm's law are called Non – Ohmic conductors. So V-I grapis not linear.

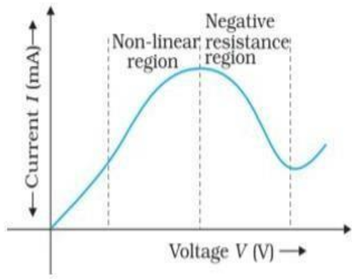
Eg:- Semi conductors, Diodes , Transistors.

The deviations broadly are one or more of the following types:

a) The value of V stops to be proportional to I.
b) The value of current changes when we reverse the direction of V.
c) The relation between V and I is not unique, i.e., there is more than one value of V for the same current I.







3.7 Resistivity of Various Materials

The materials are classified as conductors, semiconductors and insulators depending on their resistivities, in an increasing order of their values.

- Metals have low resistivities in the range of 10^{-8} Ωm to 10^{-6} Ωm.
- Insulators like ceramic, rubber and plastics having resistivities 10^{18} times greater than metals or more.
- In between the two are the semiconductors

3.8 Temperature Dependence Of Resistivity

The resistivity of a material is found to be dependent on the temperature.

The resistivity of a metallic conductor is approximately given by,

$$\boldsymbol{\rho_T = \rho_0[1 + a(T - T_0)]}$$

Where ρ_0 is the resistivity at a reference temperature T_0. ρ_T is the resistivity at a temperature T

α is called the temperature co-efficient of resistivity

$$\frac{\rho_T}{\rho_0} = [1 + \alpha(T - T_0)]$$

$$\frac{\rho_T}{\rho_0} - 1 = \alpha(T - T_0)$$

$$\frac{\rho_T - \rho_0}{\rho_0} = \alpha(T - T_0)$$

$$a = \frac{\rho_T - \rho_0}{\rho_0(T - T_0)}$$

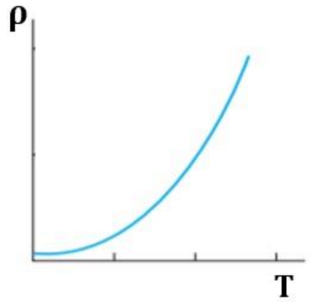
The dimension of α is $[\text{Temperature}]^{-1}$ and unit is K^{-1}.

When temp increases, if the resistivity increases, then α is positive

When temp increases, if the resistivity decreases, then α is negative

For metals

For metals, α is positive ie, when temp increases, the resistivity also increases. $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$ $\rho \propto \frac{1}{\tau}$

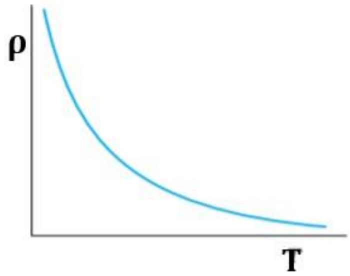


When temperature increases, the collisions of free electrons increases, relaxation time decreases and hence the resistivity increases.

For insulators and semiconductors,

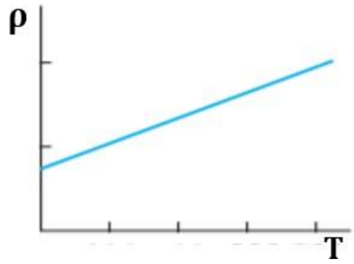
For insulators and semiconductors , α is negative ie., when temp increases, the resistivity decreases.

$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$ $\rho \propto \frac{1}{n}$



When temp increases, the number n of free electrons per unit volume increases , and hence the resistivity decreases.

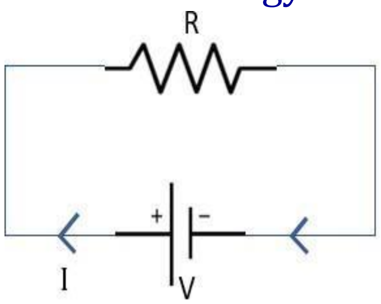
For Nichrome ,Manganin and constantan



Nichrome (which is an alloy of nickel, iron and chromium), Manganin and constantan exhibit a very weak dependence of resistivity with temperature. These materials are thus widely used in wire bound standard resistors since their resistance values would change very little with temperatures.

3.9 Electrical Energy, Power

Electrical Energy



Work done by the cell on a charge q under a pd of V volt

$W = qV$

$q = It$

$W = VIt$

This is work is same as the electricl energy supplied by the cell to the charges

$$E = VIt$$

The electrons move with increased KE and make collisions with atoms. The energy gained by the charges is shared with the atoms. The atoms vibrate more vigorously, i.e., the conductor heats up.

The amount of energy dissipated as heat in the conductor during the time interval t is

$$E = VIt$$

Using Ohm's law V = IR,

$E = VI\,t$

$E = IR\,xIt$

$E = I^2Rt$

$I = \frac{V}{R}$

$E = VI\,t = V\ x\frac{V}{R}t$

$E = \frac{V^2t}{R}$

Power

Power is the energy dissipated per unit time

$P = \frac{E}{t}$

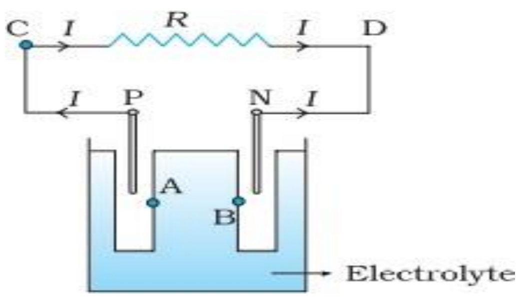
Unit of power is watt(W)

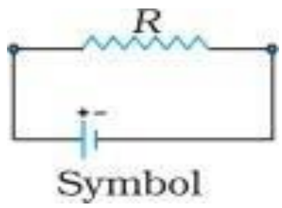
$E = VI\,t$	$E = \frac{V^2t}{R}$	$E = I^2Rt$
$P = VIt/t$	$P = \frac{V^2t}{R}/t$	$P = \frac{I^2Rt}{t}$
$P = VI$	$P = \frac{V^2}{R}$	$P = I^2R$

The power loss or "ohmic loss" in a conductor of resistance R carrying a current I is given by these equations. It is this power which heats up, the coil of an electric bulb to incandescence, radiating out heat and light.The external source ,that is the cell supplies this power.The chemical energy of the cell supplies this power for as long as it can.

3.10 Cells, Emf, Internal Resistance

Cell





A simple device which maintain a steady current in an electric circuit is the electrolytic cell.

Basically a cell has two electrodes, called the positive (P) and the negative(N) .They are immersed in an electrolytic solution. The electrodes exchange charges with the electrolyte.

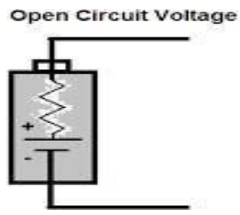
Internal resistance of a cell (r)

Resistance offered by the electrolytes to the flow of current through it is called internal resistance of the cell

E.M.F -Electro Motive Force (ε)

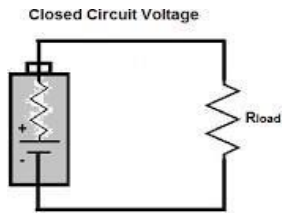
The emf ε is the potential difference between the positive and negative electrodes of a cell in an open circuit, i.e., when no current is flowing through the cell.

Note that ε is, actually, a potential difference and not a force.

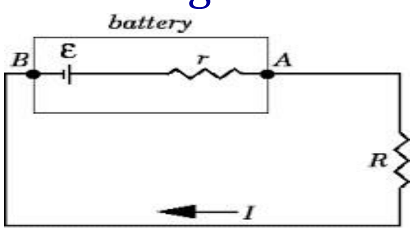


Voltage (V)

The Voltage (V) is the potential difference between the positive and negative electrodes of a cell in a closed circuit, i.e., when current is flowing through the cell.



Relation connecting emf and Voltage



$$\text{Current } I = \frac{\text{emf}}{\text{Total Resistance}}$$

$$\mathbf{I = \frac{\varepsilon}{R+r}}$$

$$\varepsilon = I\,(R + r)$$

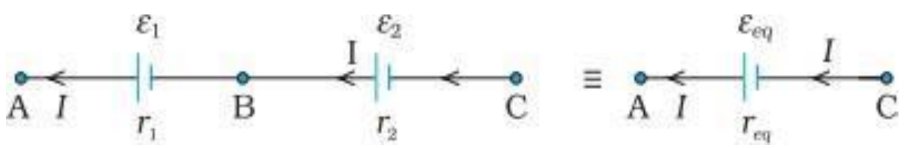
$$\varepsilon = IR + Ir$$

$$\varepsilon = V + Ir$$

$$\mathbf{V = \varepsilon - Ir}$$

3.11 Cells in Series and Parallel Cells

in Series



$$V_{AC} = \varepsilon_{eq} - I\, r_{eq}$$

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$$

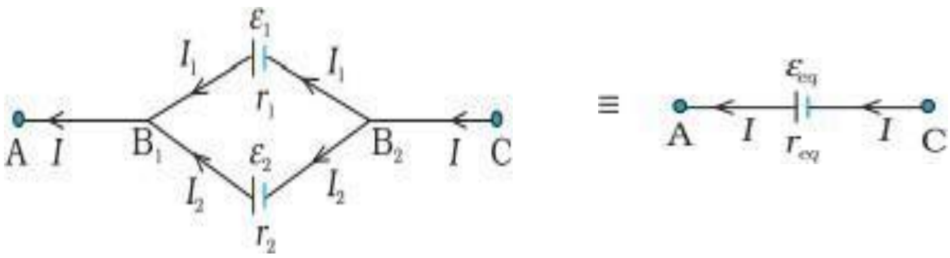
$$\mathbf{r_{eq} = r_1 + r_2}$$

For n cells in series,

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + \dots\dots\dots + \varepsilon_n$$

$$r_{eq} = r_1 + r_2 + \dots\ \dots\dots\dots + r_n$$

Cells in parallel



$$V_{AC} = \varepsilon_{eq} - I\, r_{eq}$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\frac{s_{eq}}{r_{eq}} = \frac{s_1}{r_1} + \frac{s_2}{r_2}$$

For n cells in parallel

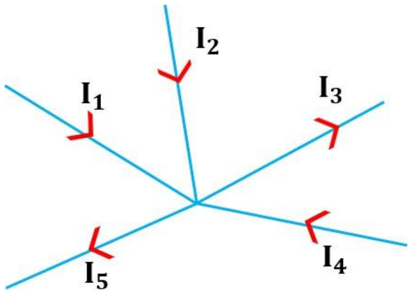
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots\dots\dots\dots + \frac{1}{r_n}$$

$$\frac{s_{eq}}{r_{eq}} = \frac{s_1}{r_1} + \frac{s_2}{r_2} + \dots\ \dots\dots\dots + \frac{s_n}{r_n}$$

3.12 Kirchhoff's Rules

(a) Kirchhoff's First Rule - Junction Rule:

At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction . $\sum I = 0$.

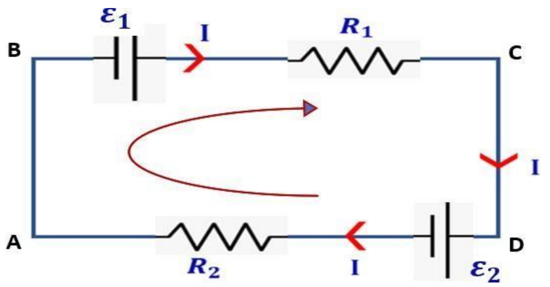


$$I_1 + I_2 + I_4 = I_3 + I_5$$
$$I_1 + I_2 - I_3 + I_4 - I_5 = 0$$
$$\sum I = 0$$

Kirchhoff's junction rule is in accordance with law of conservation of charge.

(b) Kirchhoff's Second Rule –Loop Rule

The algebraic sum of changes in potential around any closed loop is zero.
$\sum \Delta V = 0$

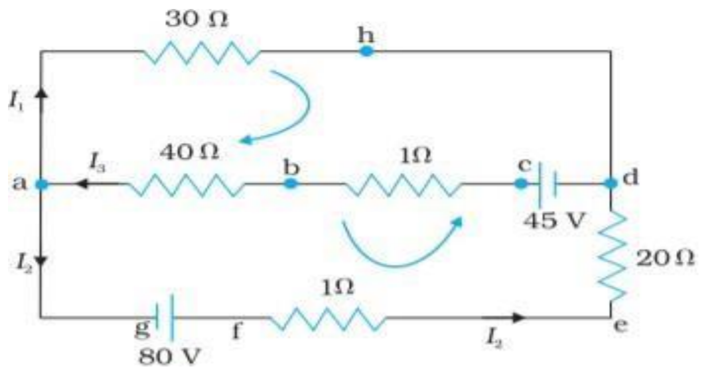


For Loop ABCDA $\varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$

For Cell
If Path from –ve to +ve terminal, $\Delta V = +\varepsilon$
If Path from +ve to -ve terminal $\Delta V = -\varepsilon$
For resistor
If path is in same direction of current $\Delta V = -IR$
If path is in opposite direction of current $\Delta V = +IR$

Kirchhoff's Loop rule is in accordance with Law of conservation of energy.

Example



Applying Junction rule at junction 'a'

$$I_3 = I_1 + I_2$$

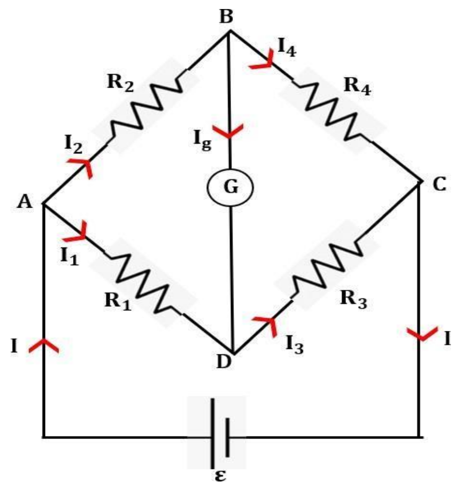
Applying Loop rule for the loops 'ahdcba' ,

$$-30\ I_1 + 45 - 1\ I_3 - 40\ I_3 = 0$$

Applying Loop rule for the loop 'ahdefga',

$$-30\ I_1 + 20\ I_2 + 1\ I_2 - 80 = 0.$$

3.13 Wheatstone Bridge



For a balanced Wheatstone's bridge , the resistors are such that the current through th galvanometer $I_g = 0$. Apply Kirchhoff's junction rule to junctions B

$$\mathbf{I_2 = I_4} \text{-------------(1)}$$

Apply Kirchhoff's junction rule to junctions D

$$\mathbf{I_1 = I_3} \text{-------------(2)}$$

Apply Kirchhoff's loop rule to closed loop ABDA

$$\mathbf{I_1 R_1 = I_2 R_2} \text{----------------(3)}$$

Apply Kirchhoff's loop rule to closed loop CBDC

$$I_3 R_3 = I_4 R_4 \text{ ---------------(4)}$$

eq (3) -------- $\frac{I_1 R_1}{} = \frac{}{I_2 R_2}$

eq (4) $I_3 R_3 \quad I_4 R_4$

$$\frac{R_1}{R_3} \frac{R_2}{R_4} = \_$$

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

This is the balance condition for the galvanometer to give zero or null deflection.