3. MATRIX ALGEBRA

1. Matrix was first introduced by Arthur Cayley in 1858. It was first used for the study of linear equations and linear transformations. Now it is largely used in disciplines like Physics, Chemistry, Statistics, and Engineering etc.
2. Matrix is an array of number arranged in rows and columns.
3. The numbers constituting a matrix is known as elements /members.
4. Matrices are denoted by capital letters of English alphabet and elements are denoted by small letters.
5. If a matrix has 'm' rows and 'n' columns its order is m x n (read as 'm by n').

   E.g.: - $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}$ $\quad B = \begin{bmatrix} 0 & -2 \\ 1 & 5 \end{bmatrix}_{2\times2}$

6. In general, an $m\times n$ matrix is written as:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}$$

Here (a_{ij}) is the general element where

$i = 1,2,\ldots\ldots,m$ and

$j = 1,2,3, \ldots\ldots ,n$.

Note: The double subscript 'ij' is called the address of the element.

Types of matrices

1. **Square Matrix**: number of rows = number of columns.
2. **Zero Matrix**: all elements are zeroes. It is denoted by 'O'.
3. **Diagonal Matrix**: a square matrix, having non-zero diagonal elements on the main diagonal.
4. **Scalar Matrix**: In a diagonal matrix, diagonal elements are same.
5. **Identity Matrix**: In a diagonal matrix, diagonal elements are unity. It is denoted by 'I'.

   $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, etc.

6. **Equality of Matrices**: Two matrices are said to be equal if (a) they are of the same order (b) each elements of A = corresponding element of B. i.e., $(a_{ij} = b_{ij})$

7. **Upper Triangular Matrix**: In a square matrix, all elements below the diagonal elements are zeroes.
8. **Lower Triangular Matrix**: In a square matrix, all elements above the diagonal elements are zeroes.
9. **Row matrix**: Matrix having only one row.
10. **Column Matrix**: Matrix having only one column.

Addition of matrices

Two matrices are conformable for addition if and only if they are of the same order. The sum matrix is got by adding the corresponding elements of both the matrices.

i.e., $\left[a_{ij}\right]+\left[b_{ij}\right]=\left[a_{ij}+b_{ij}\right]$, where a_{ij} and b_{ij} are matrices of the same order.

Scalar Multiplication of a matrix

Let A be a $m\times n$ matrix and 'm' be a scalar (number). Then the scalar multiple of a matrix is obtained by multiplying all the elements of A by the scalar 'm'. i.e., if $A=\left[a_{ij}\right]\Rightarrow kA=\left[ka_{ij}\right]$.

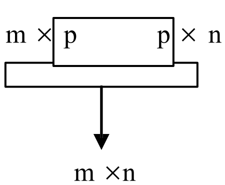
Multiplication of matrices

Two matrices are conformable for multiplication if and only if the number of columns of 1st matrix = the number of rows of 2nd matrix. The 1st element in the 1st row of the product matrix is obtained by taking the sum of the product of the corresponding elements of the 1st row of the 1st matrix to the corresponding elements of the 1st column of the 2nd matrix. The 2nd element in the 1st row of the product matrix is obtained by taking the sum of the product of the corresponding elements of the 1st row of the 1st matrix to the corresponding elements of the 2nd column of the 2nd matrix and so on.

$$\text{Let } A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} \qquad B=\begin{bmatrix}2 & 3\\ 4 & 5\\ 1 & 2\end{bmatrix}$$

$$AB=\begin{bmatrix}1\times 2+2\times 4+3\times 1 & 1\times 3+2\times 5+3\times 2\\ 4\times 2+5\times 4+6\times 1 & 4\times 3+5\times 5+6\times 2\end{bmatrix}=\begin{bmatrix}2+8+3 & 3+10+6\\ 8+20+6 & 12+25+12\end{bmatrix}=\begin{bmatrix}13 & 19\\ 34 & 49\end{bmatrix}$$

Note : If matrix A is of order $m\times n$ and B is of order $p\times n$ then



AB is possible and is of order $m\times n$.

Matrix polynomial

If $f(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\quad +a_3x^3+a_2x^2+a_1x+a_0$ be a polynomial function and A be a square matrix of order 'n', then $f(A)=a_nA^n+a_{n-1}A^{n-1}+a_{n-2}A^{n-2}+\quad +a_3A^3+a_2A^2+a_1A+a_0$ is known as matrix polynomial.

Note1: $A=\begin{bmatrix}1&3\\2&4\end{bmatrix}$

$$then\ A^2=A.A=\begin{bmatrix}1&3\\2&4\end{bmatrix}\begin{bmatrix}1&3\\2&4\end{bmatrix}=\begin{bmatrix}1+6&3+12\\2+8&6+16\end{bmatrix}=\begin{bmatrix}7&15\\10&22\end{bmatrix}$$

Note2:

i. If $A^2=A$, then A is known as an idempotent matrix.

ii. If $A^2=I$, then A is known as an involuntary matrix

iii. If $A^2=0$, then A is known as a nilpotent matrix

E.g.: If A = $\begin{bmatrix}3&-2\\4&-2\end{bmatrix}$, and I=$\begin{bmatrix}1&0\\0&1\end{bmatrix}$ find k so that $A^2=kA-2I$.

$$A^2=\begin{bmatrix}3&-2\\4&-2\end{bmatrix}\begin{bmatrix}3&-2\\4&-2\end{bmatrix}=\begin{bmatrix}9+-8&-6+4\\12-8&-8+4\end{bmatrix}=\begin{bmatrix}1&-2\\4&-4\end{bmatrix}$$

$$kA=k\begin{bmatrix}3&-2\\4&-2\end{bmatrix}=\begin{bmatrix}3k&-2k\\4k&-2k\end{bmatrix}$$

$$2I=\begin{bmatrix}2&0\\0&2\end{bmatrix}$$

$$A^2+2I=kA\Rightarrow kA=A^2+2I\Rightarrow\begin{bmatrix}3k&-2k\\4k&-2k\end{bmatrix}=\begin{bmatrix}1&-2\\4&-4\end{bmatrix}+\begin{bmatrix}2&0\\0&2\end{bmatrix}$$

$$\begin{bmatrix}3k&-2k\\4k&-2k\end{bmatrix}=\begin{bmatrix}3&-2\\4&-2\end{bmatrix}\Rightarrow k=1$$

Transpose of a matrix

Interchange of rows and columns of a matrix is known as transpose matrix.

A $=\begin{bmatrix}2&3&1\\3&2&5\end{bmatrix}$, then A transpose, $A^T\ or\ A'=\begin{bmatrix}2&3\\3&2\\1&5\end{bmatrix}$

Theorem: If A^T and B^T be the transposes of the matrices A and B respectively, then

- $\left(A^T\right)^T = A$
- $(kA)^T = k.A^T$
- $(A+B)^T = A^T + B^T$
- $(AB)^T = B^T.A^T$

Symmetric & Skew Symmetric Matrices

A square matrix A is said to be symmetric if $A^T = A$ and skew-symmetric if $A^T = -A$.

Note: The diagonal elements of a skew-symmetric matrix are zeroes.

Tips ☞

- If A and B are symmetric matrices, $(AB)^T = AB$, provided $AB = BA$
- If A is any square matrix, then $A + A^T$ is symmetric and $A - A^T$ is skew-symmetric.
- If A is any square matrix, then AA^T *and* $A^T A$ both are symmetric matrices.
- If A is a symmetric matrix, then A^n is also symmetric.
- If A and B are symmetric matrices, $AB + BA$ is symmetric.
- If A and B are symmetric matrices of same order, then $AB - BA$ is symmetric.
- Every square matrix can be expressed the sum of two matrices of which one is symmetric and the other is skew-symmetric.

Elementary transformations

There are 6 elementary transformations – 3 for row transformations and 3 for column transformations.

1. Interchanging a pair of rows or columns:

   $R_i \leftrightarrow R_j$; $C_i \leftrightarrow C_j$

   E.g.: i) $\begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$ $R_1 \leftrightarrow R_2$

   ii) $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ $C_1 \leftrightarrow C_2$

2. Multiplying each element of a row or column by a non-zero number:

   $R_i \to kR_i$; $C_i \to kC_i$, k is any scalar.

E.g.: i) $\begin{bmatrix} 2 & 6 \\ 4 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \qquad R_1 \rightarrow \frac{1}{2} R_1$

ii) $\begin{bmatrix} 3 & 0 \\ 6 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} \qquad C_1 \rightarrow \frac{1}{3} C_1$

3. To each element of a row or column of a matrix, a multiple of another row or column is added,

$R_i \rightarrow R_i + kR_j \quad ; C_i \rightarrow C_i + kC_j$

E.g.: i) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} \qquad R_2 \rightarrow R_2 - 3R_1$

ii) $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix} \qquad C_2 \rightarrow C_2 - 2C_1$

PROBLEMS

1. In the matrix $\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write:

(i) the order of the matrix (ii) the number of elements,

(iii) the elements a_{13}, a_{21}, a_{33}, a_{24}, a_{23}

(i) In the given matrix, the number of rows is 3 and the number of columns is 4. Therefore, the order of the matrix is **3 × 4**.

(ii) Since the order of the matrix is 3 × 4, there are 3 × 4 = **12** elements in it.

(iii) $a_{13} = 19$, $a_{21} = 35$, $a_{33} = -5$, $a_{24} = 12$, $a_{23} = \frac{5}{2}$

2. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

We know that if a matrix is of the order $m \times n$, it has mn elements. The factors of 24 are: 1 × 24, 2 × 12, 3 × 8 and 4 × 6. Hence, the possible orders of a matrix having 24 elements are: 1 × 24, 24 × 1, 2 × 12, 12 × 2, 3 × 8, 8 × 3, 4 × 6, and 6 × 4

If it has 13 elements, the factors are 1 × 13 (13 is a prime number). Hence, the possible orders of a matrix having 13 elements are 1 × 13 and 13 × 1.

3. Construct a 3 × 4 matrix, whose elements are given by (i) $a_{ij} = \frac{1}{2}|-3i + j|$ ii) $a_{ij} = \frac{1}{2}|2i - j|$

In general, a 3 × 4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

(i) $a_{ij} = \frac{1}{2}|-3i + j|,\ i = 1,2,3\ ;\ j = 1,2,3,4$

$$a_{11} = \frac{1}{2}|-3+1| = \frac{1}{2}|-2| = 1$$

$$a_{12} = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}$$

$$a_{13} = \frac{1}{2}|-3+3| = \frac{1}{2}|0| = 0$$

$$a_{14} = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}$$

$$a_{21} = \frac{1}{2}|-6+1| = \frac{1}{2}|-5| = \frac{5}{2}$$

$$a_{22} = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = 2$$

$$a_{23} = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{3}{2}$$

$$a_{24} = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = 1$$

$$a_{31} = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = 4$$

$$a_{32} = \frac{1}{2}|-9+2| = \frac{1}{2}|-7| = \frac{7}{2}$$

$$a_{33} = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = 3$$

$$a_{34} = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = -\frac{5}{2}$$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

(ii) $a_{ij} = 2i - j,\ i = 1,2,3\ ;\ j = 1,2,3,4$

$$a_{11} = 2\times1 - 1 = 2 - 1 = 1$$

$$a_{12} = 2\times1 - 2 = 2 - 2 = 0$$

$a_{13} = 2\times1-3 = 2-3 = -1$

$a_{14} = 2\times1-4 = 2-4 = -2$

$a_{21} = 2\times2-1 = 4-1 = 3$

$a_{22} = 2\times2-2 = 4-2 = 2$

$a_{23} = 2\times2-3 = 4-3 = 1$

$a_{24} = 2\times2-4 = 4-4 = 0$

$a_{31} = 2\times3-1 = 6-1 = 5$

$a_{32} = 2\times3-2 = 6-2 = 4$

$a_{33} = 2\times3-3 = 6-3 = 3$

$a_{34} = 2\times3-4 = 6-4 = 2$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

4. Find the value of x, y, and z from the following equation:

i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

If the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get: $x = 1$, $y = 4$, and $z = 3$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

If the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$x + y = 6$...................... (1)

$xy = 8$

$5 + z = 5 \Rightarrow z = 5-5 = 0$

We know that: $(x-y)^2 = (x+y)^2 - 4xy \Rightarrow (x-y)^2 = (6)^2 - 4\times8 \Rightarrow (x-y)^2 = 36-32 = 4$

$\Rightarrow x - y = \pm2$.................... (2)

Now, when $x - y = 2$ and $x + y = 6$, we get $x = 4$ and $y = 2$

When $x - y = -2$ and $x + y = 6$, we get $x = 2$ and $y = 4$

$\therefore x = 4$, $y = 2$, and $z = 0$ or $x = 2$, $y = 4$, and $z = 0$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

If two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$x + y + z = 9$...................... (1)

$x + z = 5$.............................. (2)

$y + z = 7$........................... (3)

From (1) and (2), we have:

$y + 5 = 9 \Rightarrow y = 4$

Then, from (3), we have:

$4 + z = 7 \Rightarrow z = 3$

$\therefore x + z = 5 \Rightarrow x = 2$

$\therefore x = 2$, $y = 4$, and $z = 3$

5. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:

(A) 27 (B) 18 (C) 81 (D) 512

Number of Matrices $= (No.of entries)^{mn} = 2^{3\times 3} = 2^9 = 512$

The answer is D.

6. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = x^2 - 2x - 3$, find f(A)

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ then $f(A) = A^2 - 2A - 3I$,

$$A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}; \; 3I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}.$$

$$f(A) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

7. If A= $\begin{bmatrix} 5 & 3 & 10 \end{bmatrix}$ *and* $B = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ then find AB

$$AB = \begin{bmatrix} 5 & 3 & 10 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = [5\times2+3\times4+10\times6] = [10+12+60] = [82]$$

8. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 5 & 4 \\ 1 & 6 \end{bmatrix}$

a) What is the order of AB?

b) Find A^T and B^T.

c) Verify that $(AB)^T = B^T A^T$

a) order of AB is 2 x 2 (1)

b) $A^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -3 & -1 \end{bmatrix}$ and $B^T = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 4 & 6 \end{bmatrix}$ (2)

c) $AB = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 5 & 4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 9 & -7 \\ 8 & 4 \end{bmatrix}$

$$(AB)^T = \begin{bmatrix} 9 & 8 \\ -7 & 4 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$B^T A^T = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 4 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ -7 & 4 \end{bmatrix} \ldots\ldots\ldots\ldots (2)$$

From (1) and (2) we have $(AB)^T = B^T A^T$.

9. Consider the following statement:

$P(n): A = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for all n∈N

a) Write P(1).

b) If P(k) is true, then show that P(k+1) is true.

c) Show that P(n) is true for all positive integral values of n ∈N.

. $P(n): A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

$$P(1): A^1 = \begin{bmatrix} 1+2\times1 & -4\times1 \\ 1 & 1-2\times1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

∴ P(1) is true.

Assume that P(k) be true.

$$P(k): A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$$

To prove that P(k+1) is true.

$$P(k+1): A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -1-2k \end{bmatrix}$$

$$\text{LHS} = A^{k+1} = A^k . A = A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+2k)3+(-4k)1 & (1+2k)\times -4 + -4k \times -1 \\ 3k+1-2k & -4k + -1(1-2k) \end{bmatrix}$$

$$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ k+1 & -4k-1+2k \end{bmatrix} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -1-2k \end{bmatrix} = \text{RHS.}$$

Hence P(k+1) is true.

Hence P(n) is true for all values of $n \in N$.

10. The book shop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs.80, Rs.60 and Rs.40 respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Let $A = \begin{bmatrix} 120 & 96 & 120 \end{bmatrix} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix} = \begin{bmatrix} 120\times 80 + 96\times 60 + 120\times 40 \end{bmatrix}$

$= \begin{bmatrix} 9600+5760+4800 \end{bmatrix} = 20160$

The total amount the bookshop will receive is Rs.20,160.

11. Using elementary transformation, find the inverse of $\begin{bmatrix} 1 & 5 \\ 3 & 4 \end{bmatrix}$. (3)

$$A = IA \Rightarrow \begin{bmatrix} 1 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} 1 & 5 \\ 0 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} A$$

$R_2 \to \frac{1}{-11} R_2$

$$\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{11} & -\frac{1}{11} \end{bmatrix} A$$

$R_1 \to R_1 - 5R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{4}{11} & \frac{5}{11} \\ \frac{3}{11} & -\frac{1}{11} \end{bmatrix} A \Rightarrow A^{-1} = \frac{1}{11} \begin{bmatrix} -4 & 5 \\ 3 & -1 \end{bmatrix}$$