

SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2023

PART III

SUBJECT: STATISTICS

CODE NO: **SY- 532**

VERSION:

SCORES: **60**

2 HOURS

Qn, No.	Sub Qns	Answer Key / Value Points	Score	Total Score
1		Explanation/graphical presentation of three types of correlation- Positive, Negative, Zero (for writing names only give $\frac{1}{2}$ score each)	1+1+1	3
2	(a)	(iii)perpendicular OR (iv) intersect	1	3
	(b)	$b_{xy} = r \frac{\sigma_x}{\sigma_y}$ $1.5 = \frac{0.65 \times 4}{\sigma_y} \Rightarrow \sigma_y = 1.73$	$\frac{1}{2} + \frac{1}{2}$	
3		$E(X) = \sum xP(x)$ $= 0 \times 0.1 + 1 \times 0.3 + 2 \times 0.4 + 3 \times 0.2 = 1.7$ $E(X^2) = \sum x^2 P(x)$ $= 0^2 \times 0.1 + 1^2 \times 0.3 + 2^2 \times 0.4 + 3^2 \times 0.2 = 3.7$ $V(X) = E(X^2) - (E(X))^2$ $= 3.7 - 1.7^2 = 3.7 - 2.89 = 0.81$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	3
4	(a)	Definition of cdf.	1	3
	(b)	Any two properties of cdf (1 score each)	2	
5	(a)	(iv) 0.5	1	3
	(b)	Given mean $\mu = 28$, standard deviation $\sigma = 5$ $P(25 < X < 30) = P\left(\frac{25-28}{5} < \frac{X-28}{5} < \frac{30-28}{5}\right)$ $= P(-0.6 < Z < 0.4)$ $= P(0 < Z < 0.6) + P(0 < Z < 0.4)$ $= 0.2257 + 0.1554 = 0.3811$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	
6	(a)	(ii) 2	1	3
	(b)	Any four properties of normal curve ($\frac{1}{2}$ score each)	$4 \times \frac{1}{2} = 2$	
7		$V(T_1) = V(2X_1 + 2X_2 - 3X_3) = 4V(X_1) + 4V(X_2) + 9V(X_3) = 4\sigma^2 + 4\sigma^2 + 9\sigma^2 = 17\sigma^2$ $V(T_2) = V(X_1 + 2X_2 - 2X_3) = V(X_1) + 4V(X_2) + 4V(X_3) = \sigma^2 + 4\sigma^2 + 4\sigma^2 = 9\sigma^2$ Here $V(T_2) < V(T_1)$. Hence T_2 the efficient estimator.	1 1 1	3

Qn, No.	Sub Qns	Answer Key / Value Points	Score	Total Score																						
8	(a)	(i) unbiased	1	3																						
	(b)	(ii) Moment estimate of $\mu = \bar{x}$ $= \frac{\sum x}{n} = \frac{201}{12} = 16.75$	1 $\frac{1}{2} + \frac{1}{2}$																							
9	(a)	Definition or Explanation of assignable causes	1	3																						
	(b)	Normality, homogeneity, independence, additivity	$4 \times \frac{1}{2} = 2$																							
10		<table border="1"><thead><tr><th>Source</th><th>df</th><th>SS</th><th>MSS</th><th>F</th><th>F_α</th></tr></thead><tbody><tr><td>Between</td><td>8</td><td><u>361.6</u></td><td>45.2</td><td rowspan="2"><u>12.91</u></td><td rowspan="2">2.95</td></tr><tr><td>Within</td><td><u>11</u></td><td><u>38.5</u></td><td>3.5</td></tr><tr><td>Total</td><td>19</td><td>400.1</td><td></td><td></td><td></td></tr></tbody></table> $(4 \times \frac{1}{2})$ $F > F_{\alpha}$ so we reject null hypothesis	Source	df	SS	MSS	F	F _α	Between	8	<u>361.6</u>	45.2	<u>12.91</u>	2.95	Within	<u>11</u>	<u>38.5</u>	3.5	Total	19	400.1				2 $\frac{1}{2} + \frac{1}{2}$	3
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11	(a)	(iv) irregular variation	1	3																						
	(b)	Explanation of cyclical variation with or without diagram OR Definition of cyclical variation	2																							
12	(a)	(iv) current year quantity	1	3																						
	(b)	Any four uses of index numbers.	$4 \times \frac{1}{2} = 2$																							
13	(a)	Let $2x + 3y - 31 = 0$ ---- (1), $5x + 4y - 28 = 0$ ---- (2) Let us assume that eqn (1) is the regression line of Y on X and eqn (2) is the regression line of X on Y. Now eqn (1) becomes, $3y = -2x + 31 \Rightarrow y = \frac{-2}{3}x + \frac{31}{3} \therefore b_{yx} = \frac{-2}{3}$ Similarly eqn (2) becomes, $5x = -4y + 28 \Rightarrow x = \frac{-4}{5}y + \frac{28}{5} \therefore b_{xy} = \frac{-4}{5}$ Now $b_{yx} \times b_{xy} = \frac{-2}{3} \times \frac{-4}{5} = \frac{8}{15} < 1$ Our assumption is right. So the regression line of Y on X is $2x + 3y - 31 = 0$	1 1 $\frac{1}{2}$ $\frac{1}{2}$	4																						
		(b)	$r = \pm \sqrt{b_{yx} \times b_{xy}}$ $= \pm \sqrt{\frac{8}{15}} = \pm \sqrt{0.53} = -0.73$		$\frac{1}{2}$ $\frac{1}{2}$																					

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14	(a)	$\frac{dy}{dx} = 12x^2 - 4x + 7$ $\frac{d^2y}{dx^2} = 24x - 4$	1 1	4																																				
	(b)	$\int_0^2 (x^2 - 2)dx = \left[\frac{x^3}{3} \right]_0^2 - [2x]_0^2$ $= \left(\frac{2^3}{3} - 0 \right) - (4 - 0)$ $= \frac{8}{3} - 4 = \frac{-4}{3} = -1.33$	1 $\frac{1}{2}$ $\frac{1}{2}$																																					
15	(a)	(ii) Poisson distribution	1	4																																				
	(b)	$np = 4, npq = 3$ $npq = 3 \Rightarrow 4 \times q = 3 \Rightarrow q = \frac{3}{4}$ $\therefore p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$ Now, $np = 4 \Rightarrow n \times \frac{1}{4} = 4 \Rightarrow n = 4 \times 4 = 16$ The pdf is, $P(x) = {}^nC_x p^x q^{n-x}, x = 0, 1, 2, \dots, n$ $= {}^{16}C_x \left(\frac{1}{4} \right)^x \left(\frac{3}{4} \right)^{16-x}, x = 0, 1, 2, \dots, 16$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$																																					
16	(a)	(i) $\chi^2_{(1)}$	1	4																																				
	(b)	<table border="1"><thead><tr><th>Sl No</th><th>Sample</th><th>Sample Mean</th></tr></thead><tbody><tr><td>1</td><td>3,5</td><td>4</td></tr><tr><td>2</td><td>3,7</td><td>5</td></tr><tr><td>3</td><td>3,9</td><td>6</td></tr><tr><td>4</td><td>3,11</td><td>7</td></tr><tr><td>5</td><td>5,7</td><td>6</td></tr><tr><td>6</td><td>5,9</td><td>7</td></tr><tr><td>7</td><td>5,11</td><td>8</td></tr><tr><td>8</td><td>7,9</td><td>8</td></tr><tr><td>9</td><td>7,11</td><td>9</td></tr><tr><td>10</td><td>9,11</td><td>10</td></tr><tr><td colspan="2">Total</td><td>70</td></tr></tbody></table> $E(\bar{X}) = \frac{\sum \bar{x}}{n} = \frac{70}{10} = 7$	Sl No		Sample	Sample Mean	1	3,5	4	2	3,7	5	3	3,9	6	4	3,11	7	5	5,7	6	6	5,9	7	7	5,11	8	8	7,9	8	9	7,11	9	10	9,11	10	Total		70	2
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17	(a)	(iii) Chi – square test	1	4																																																		
	(b)	(i) Definition/Explanation of level of significance (ii) Definition/Explanation of power of a test (iii) Definition/Explanation of critical region	1 1 1																																																			
18		<table><tr><th>Year</th><th>No. of deaths</th><th>3 yearly moving total</th><th>3 yearly moving average</th></tr><tr><td>2011</td><td>175</td><td></td><td></td></tr><tr><td>2012</td><td>190</td><td>550</td><td>183.33</td></tr><tr><td>2013</td><td>185</td><td>570</td><td>190</td></tr><tr><td>2014</td><td>195</td><td>560</td><td>186.67</td></tr><tr><td>2015</td><td>180</td><td>578</td><td>192.67</td></tr><tr><td>2016</td><td>203</td><td>580</td><td>193.33</td></tr><tr><td>2017</td><td>197</td><td>652</td><td>217.33</td></tr><tr><td>2018</td><td>252</td><td></td><td></td></tr></table>	Year		No. of deaths	3 yearly moving total	3 yearly moving average	2011	175			2012	190	550	183.33	2013	185	570	190	2014	195	560	186.67	2015	180	578	192.67	2016	203	580	193.33	2017	197	652	217.33	2018	252			2+2	4													
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19		$r = \frac{n \sum XY - \sum X \sum Y}{\sqrt{n \sum X^2 - (\sum X)^2} \times \sqrt{n \sum Y^2 - (\sum Y)^2}}$ <table><tr><th>X</th><th>Y</th><th>X²</th><th>Y²</th><th>XY</th></tr><tr><td>18</td><td>12</td><td>324</td><td>144</td><td>216</td></tr><tr><td>28</td><td>19</td><td>784</td><td>361</td><td>532</td></tr><tr><td>12</td><td>21</td><td>144</td><td>441</td><td>252</td></tr><tr><td>25</td><td>34</td><td>625</td><td>1156</td><td>850</td></tr><tr><td>22</td><td>25</td><td>484</td><td>625</td><td>550</td></tr><tr><td>15</td><td>20</td><td>225</td><td>400</td><td>300</td></tr><tr><td>7</td><td>15</td><td>49</td><td>225</td><td>105</td></tr><tr><td>16</td><td>14</td><td>256</td><td>196</td><td>224</td></tr><tr><td>$\sum X = 143$</td><td>$\sum Y = 160$</td><td>$\sum X^2 = 2891$</td><td>$\sum Y^2 = 3548$</td><td>$\sum XY = 3029$</td></tr></table> $\therefore r = \frac{8 \times 3029 - 143 \times 160}{\sqrt{8 \times 2891 - (143)^2} \times \sqrt{8 \times 3548 - (160)^2}}$ $= \frac{1352}{\sqrt{2679} \times \sqrt{2784}} = 0.495$ Alternative method: [Covariance = 21.13 (1 score), s.d(x) = 6.47 ($\frac{1}{2}$ score), s.d(y) = 6.6 ($\frac{1}{2}$ score), $r = 0.495$ ($\frac{1}{2} + \frac{1}{2}$ Score)]	X	Y	X ²	Y ²	XY	18	12	324	144	216	28	19	784	361	532	12	21	144	441	252	25	34	625	1156	850	22	25	484	625	550	15	20	225	400	300	7	15	49	225	105	16	14	256	196	224	$\sum X = 143$	$\sum Y = 160$	$\sum X^2 = 2891$	$\sum Y^2 = 3548$	$\sum XY = 3029$	1 2 1 1	5
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20	(a)	$\sum \bar{x} = 434, \sum R = 59$ $\bar{\bar{x}} = \frac{\sum \bar{x}}{m} = \frac{434}{12} = 36.17 \quad \bar{R} = \frac{\sum R}{m} = \frac{59}{12} = 4.92$ The control limits of $\bar{x}$ chart are, $CL = \bar{\bar{x}} = 36.17$ $UCL = \bar{\bar{x}} + A_2 \bar{R} = 36.17 + 0.729 \times 4.92 = 39.76$ $LCL = \bar{\bar{x}} - A_2 \bar{R} = 36.17 - 0.729 \times 4.92 = 32.58$	$\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	5																																																
	(b)	Drawing $\bar{x}$ chart The process is out of control. (Give 1 score for rough diagram)	2 $\frac{1}{2}$																																																	
21		<table border="1"><thead><tr><th>p_0</th><th>q_0</th><th>p_1</th><th>q_1</th><th>$p_0 q_0$</th><th>$p_1 q_0$</th><th>$p_0 q_1$</th><th>$p_1 q_1$</th></tr></thead><tbody><tr><td>30</td><td>5</td><td>39</td><td>5</td><td>150</td><td>195</td><td>150</td><td>195</td></tr><tr><td>27</td><td>8</td><td>45</td><td>4</td><td>216</td><td>360</td><td>108</td><td>180</td></tr><tr><td>36</td><td>7</td><td>38</td><td>6</td><td>252</td><td>266</td><td>216</td><td>228</td></tr><tr><td>42</td><td>3</td><td>45</td><td>4</td><td>126</td><td>135</td><td>168</td><td>180</td></tr><tr><td colspan="4">Total</td><td>744</td><td>956</td><td>642</td><td>783</td></tr></tbody></table> (i) Laspeyre's Index No. = $\frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{956}{744} \times 100 = 128.49$ (ii) Paasche's Index No. = $\frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100 = \frac{783}{642} \times 100 = 121.96$ (iii) Fisher's Index No. = $\sqrt{L \times P} = \sqrt{128.49 \times 121.96} = 125.18$	p_0	q_0	p_1	q_1	$p_0 q_0$	$p_1 q_0$	$p_0 q_1$	$p_1 q_1$	30	5	39	5	150	195	150	195	27	8	45	4	216	360	108	180	36	7	38	6	252	266	216	228	42	3	45	4	126	135	168	180	Total				744	956	642	783	2 1 1 1	5
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