SECOND YEAR HIGHER SECONDARY EXAMINATION, MARCH 2021

MATHEMATICS (SCIENCE)

CODE NO: SY 227 60 SCORES

Qn.No	Sub Qns	Answer Key/Value Points	Score	Total Score
1		3 x 3 2 x 1 4 1 = $3-x^2=3-8$ .... $x^2=8$ .... $=\pm 2\sqrt{2}$ x **Remark** 1 score for the correct expansion of determinant in LHS and 1 score for the correct expansion of determinant in RHS.(1+1)	1+1 1	**3**

2	(i)	1 OR $C_{11} = 4$ $C_{12} = -3$ ....(1) .... -3 $C_{21} = -2$ $C_{22} = 1$ $Cofactor\ Matrix = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$ ....(½)	2	**3**
	(ii)	$adj(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ $adj\ A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ .... (½) $A.adj\ A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ $= \begin{bmatrix} 4-6 & -2+2 \\ 12-12 & -6+4 \end{bmatrix}$ .... $= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ .... **OR** $\|A\| = -2$ ....(½)	½ ½	

		$A .adj\ A=\|A\|I=\begin{bmatrix}-2 & 0 \\ 0 & -2\end{bmatrix}$ ....(½)		

3		$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$ .... 5k+1 = 10 .... $k=\frac{9}{5}$ .... **Remark** ½ score for the correct expansion of LHL and ½ score for the correct expansion of RHL. (½ + ½)	1 ½ +½ 1	**3**

4		$f(x) = x^2 + 2x - 8$ (i) f is continuous in [-4, 2] ……………………………….. (ii) f is differentiable in (-4, 2) …………………………….. (iii) f(-4) = 0 = f(2) …………………………………………. $f^{I}(x) = 2x + 2$ …………………………………………………….. By Rolle's theorem $f'(c) = 0$ …………………………………………………. $2c + 2 = 0$ $c = -1 \in (-4, 2)$ ………………………………………………… Rolle's theorem is verified.	½ ½ ½ ½ ½ ½	**3**
5		$A = \pi r^2$ …………………………………………………………… $\frac{dA}{dr} = 2\pi r$………………………………………………… $= 2\pi \times 5 = 10\ \pi cm^2/cm$………………………………	1 1 1	**3**
6		$\vec{a} \cdot \vec{b} = 7-3+56=60$…………………………………………………. $\|\vec{b}\| = \sqrt{114}$ ………………………………………………. Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}$ ……………………………… $= \frac{60}{\sqrt{114}}$………………………………	1 ½ 1 ½	**3**
7		$\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$ ………………………………………………. $\vec{N} = \hat{i} - 2\hat{j} + \hat{k}$ ……………………………………………………… Vector equation of the plane is	½ ½	**3**

		$(\vec{r}-\vec{a}).\vec{N} = 0$..	1	
		$[\vec{r}-(\hat{i}+4\hat{j}+6\hat{k})].(\hat{i}-2\hat{j}+\hat{k}) = 0$	1	
		OR		
		$(x_1, y_1, z_1) = (1,4,6)$ ....(½) a, b, c = 1, -2, 1..(½)		
		The Cartesian equation of the plane is $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$.............................(1) $1(x-1) -2(y-4) +1(z-6) =0$ ....(1) $x-2y+z+1=0$		
		Remark		
		For direct answer , give full score.		
8	(i)	**[-1,1]**		**3**
		For writing the question number as **8 or 8 (i)**, give **1 score.**		
	(ii)	$\cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$		
		$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$	½	
		$\cos^{-1}\left(\frac{-1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3} + \frac{2\pi}{6}$	½	
			½	
		$= \pi$	½	

9		Area = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ ………………………………………	1	
		$= \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$ ………………………………………	1	
		$= \lvert -15 \rvert = 15$ sq units. ………………………………………	1	
		OR		
		Area = $\frac{1}{2}$[$x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)$……………(1)		**3**

		$= \frac{1}{2}$ [-2(2+8) +3(3+1) +1((-24+2)] ………..(1)		
		$= \frac{1}{2}$ (-30) = -15 = 15 sq units. ………………..(1)		
		Remark		
		For any alternate method , give 1 score for the correct equation and full score for the correct answer.		

10		$\frac{dy}{dx} - y = \cos x$ $P = -1$, $Q = \cos x$ $I.F = e^{\int P dx} = e^{\int -1 dx} = e^{-x}$ .... Solution is : $y\,(I.F) = \int Q(I.F)\,dx$ .... $\Rightarrow y.\,e^{-x} = \int \cos x.e^{-x}\,dx$ .... $\Rightarrow y.\,e^{-x} = -\cos x.\,e^{-x} - \int(-\sin x).(-e^{-x})dx$...................... $\Rightarrow y.\,e^{-x} = \frac{e^{-x}}{2}[\sin x - \cos x] + C$............................	½ 1 ½ ½ ½	**3**
11	(i) (ii)	$3A + B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix}$ $3A = \begin{bmatrix} 9 & 12 \\ -15 & -3 \end{bmatrix}$ .... $B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix} - \begin{bmatrix} 9 & 12 \\ -15 & -3 \end{bmatrix}$ .... $= \begin{bmatrix} -7 & -4 \\ 18 & -1 \end{bmatrix}$ .... $AB = \begin{bmatrix} 3 & 4 \\ -5 & -1 \end{bmatrix} \begin{bmatrix} -7 & -4 \\ 18 & -1 \end{bmatrix}$ .... $= \begin{bmatrix} -21+72 & -12-4 \\ 35-18 & 20+1 \end{bmatrix}$	½ 1 ½ ½ 1	**4**

		= $\begin{bmatrix} 51 & -16 \end{bmatrix}$ .... 17 21 **<u>Remark</u>** For 3 correct entries in B and AB, give full score.	½	
12	(i) (ii)	3×3 **B = [1 3 -6]** −2 −6 .... 12 .... AB = 4 12 −24] 5 5 15 −30 −2 4 15] .... $(AB)^1$ = −6 12 12 −24 −30 1 4 5] .... $B^1 A^1$ = 3] [−2 5 15 −30 .... −6 −2 4 = −6 12 12 −24 **<u>Remark</u>** For **attempting** the question , give **full score (4).**	1 1 ½ 1 ½	**4**

13	(i) (ii)	$^{1}\underline{x+y}$ .... $1-xy$ (c) $\tan^{-}$ **1** **-1 < x <** $\tan^{-1}\frac{2}{11}+\frac{7}{24}$ $\frac{2}{11}\cdot\frac{7}{24}$ .... LHS = $1-\tan^{-1}\underline{48+77}$ .... = $264-14$ = $\tan^{-1}\frac{125}{250}$ .... = $\tan^{-1}\frac{1}{2}$	1 1½ ½ ½ ½	**4**

		<u>**Remark**</u> For the correct formula of $\tan^{-1}x+\tan^{-1}y$ in 13(ii), give 1 score.		

14	(i) (ii)	$2x+x\frac{dy}{dx}+y+2y\frac{dy}{dx}=0$ …………………………………………… $\frac{dy}{dx}(x+2y)=-(2x+y)$ …………………………………………… $\frac{dy}{dx}=\frac{-(2x+y)}{x+2y}$ ……………………………………………………… $x=\tan\theta$ …………………………………………………………… $\theta=\tan^{-1}x$ $y=\tan^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]$ $=\sin^{-1}\sin 2\theta = 2\theta$ …………………………………………… $= 2\tan^{-1}x$ …………………………………………………………… $\frac{dy}{dx}=2.\frac{1}{1+x^2}$ …………………………………………………………… **Remark** For **attempting** the question 14 **(ii),** give **2 scores.**	1 ½ ½ ½ ½ ½ ½	**4**
15		$f^{I}(x)=4x-3$ ………………………………………………… $f^{I}(x)=0$ …………………………………………………… $4x-3=0 \Rightarrow x=\frac{3}{4}$ …………………………………………………… $f^{I}(x)>0$ in the interval $(\frac{3}{4},\infty)$,Hence increasing ………………… $f^{I}(x)<0$ in the interval $(-\infty,\frac{3}{4})$,Hence decreasing…………….. Remark For the concept of increasing and decreasing function ($f^{I}(x)>0$,increasing and $f^{I}(x)<0$,decreasing) , 2 scores.	1 ½ ½ 1 1	**4**
16	(i) (ii)	Order 2 ………………………………………………………….. Degree 1 ………………………………………………………….. $\frac{dy}{1+y^2}=(1+x^2)dx$ …………………………………………………….	½ ½ 1	**4**

	$\int \frac{dy}{1+y^2} = \int \left(1+x^2\right) dx$ .... $\tan^{-1} y = x + \frac{x^3}{3} + C$ .... **Remark** ½ score for the correct expansion of LHS and ½ score for correct expansion of RHS.	1 ½+½	
17	Find a unit vector **perpendicular to both** the vectors . $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$ .... $= -17\hat{i} + 13\hat{j} + 7\hat{k}$ .... $\lvert \vec{a} \times \vec{b} \rvert = \sqrt{507}$ .... Required Vector = $\frac{\vec{a} \times \vec{b}}{\lvert \vec{a} \times \vec{b} \rvert} = \frac{-17\hat{i} + 13\hat{j} + 7\hat{k}}{\sqrt{507}}$ .... **Remark** For **attempting** the question, give **full score (4)**.	1 1 ½ 1 ½	**4**

18	$\vec{a}_1=\hat{i}+2\hat{j}+\hat{k}$ $\vec{b}_1=\hat{i}-\hat{j}+\hat{k}$ ……………………………………	½	
	$\vec{a}_2=2\hat{i}-\hat{j}-\hat{k}$ $\vec{b}_2=2\hat{i}+\hat{j}+2\hat{k}$ ………………………………	½	
	$\vec{a}_2-\vec{a}_1=\hat{i}-3\hat{j}-2\hat{k}$ ………………………………………………	½	
	$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 2 & 1 & 2\end{vmatrix}=-3\hat{i}+3\hat{k}$ ……………………………	½ ½ 1	
	$\left\lvert\vec{b}_1\times\vec{b}_2\right\rvert=\sqrt{18}$ ………………………………………………	½	
	S.D $=\left\lvert\dfrac{(\hat{a}_2-\hat{a}_1)\cdot(\hat{b}_1\times\hat{b}_2)}{\left\lvert\hat{b}_1\times\hat{b}_2\right\rvert}\right\rvert$ ……………………………………		
	$=\left\lvert\dfrac{(\hat{i}-3\hat{j}-2\hat{k})\cdot(-3\hat{i}+3\hat{k})}{\sqrt{18}}\right\rvert=\left\lvert\dfrac{-9}{\sqrt{18}}\right\rvert=\dfrac{3}{\sqrt{2}}$ ……………		**4**

		OR $x_1=1,\ y_1=2,\ z_1=1\ a_1=1, b_1=-1, c_1=1$ ....(½) $x_2=2,\ y_2=-1,\ z_2=-1$ $a_2=2, b_2=1, c_2=2$ ....(½) $$S.D = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$$ ...(1) $$= \frac{\begin{vmatrix} 1 & -3 & -2 \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix}}{\sqrt{9+0+9}}$$ ....(1) $$= \left\lvert \frac{-9}{\sqrt{18}} \right\rvert = \frac{9}{\sqrt{18}} = \frac{3}{\sqrt{2}}$$ ....(1)		
19	(i) (ii) (iii)	$P(B/A) = \frac{P(A\cap B)}{P(A)}$ .... $P(A\cap B) = P(B/A)\times P(A)$ .... $= 0.4 \times 0.8 = 0.32$ .... $P(A/B) = \frac{P(A\cap B)}{P(B)}$ .... $= 0.32/0.5 = 0.64$ .... $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ .... $= 0.8 + 0.5 - 0.32 = 0.98$ .... **Remark**	1 ½ ½ ½ ½ ½ ½	**4**

		For $P(A/B) = \frac{P(A \cap B)}{P(B)}$ in 19(i) , give 1 score.		
20	(i)	(b) (3,1)	1	
	(ii)	R = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,2),(1,3),(1,4),(1,5),(1,6),(2,4),(2,6),(3,6)}	1	
		$(x, x) \in R, \forall x \in A$, hence reflexive	½	
		$(1,3) \in R$, but $(3,1) \notin R$, hence not symmetric	1	**4**

		$(x, y) \in R, (y, z) \in R \Rightarrow (x, z) \in R, \forall x, y, z \in A$,hence transitive	½	
		Remark For any counter example give one score to prove it is not symmetric		

21	(i) (ii)	thm on both sides $\frac{1}{y}\frac{dy}{dx}$ · =x· y dx ɔg x .... $\frac{1}{x}+\log x\cdot 1$ $\frac{dy}{dx}$ =x^x (1 dx .... +log x) $\frac{dx}{dt}$ =4at dt .... $\frac{dy}{dt}$ =4 at^3 dt .... dy .... t $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ .... = 4c 2 3 = 4 t .... at	½ 1 ½ ½ ½ ½ ½	4
22		$\frac{x}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}$ .... A=-1 , B=2 .... $\int\frac{x}{(x+1)(x+2)}dx = \int\frac{-1}{x+1}+\int\frac{2}{x+2}$ .... $=-\log\|x+1\|+2\log\|x+2\|$ .... **OR** x $\frac{x}{(x+1)(x+2)}$ = 2 x +3 x+2 x= A(2 x +3)+ B ....(½) $\frac{1}{2}$ $\frac{-3}{2}$ A=,B=...(½)	1 1 1 1	4

		$\int \frac{x}{(x+1)(x+2)}dx = \frac{1}{2}\int \frac{2x+3}{x^2+3x+2} + \frac{-3}{2}\int \frac{1}{x^2+3x+2}$ …….(½ $= \frac{1}{2}\log\lvert x^2+3x+2\rvert - \frac{3}{2}\int \frac{1}{(x+\frac{3}{2})^2-(\frac{1}{2})^2}$ ... $= \frac{1}{2}\log\lvert x^2+3x+2\rvert + \frac{3}{2}\int \frac{1}{(\frac{1}{2})^2-(x+\frac{3}{2})^2}$.($= \frac{1}{2}\log\lvert x^2+3x+2\rvert + \frac{3}{2}\log\left\lvert \frac{x+2}{-x-1}\right\rvert$ ……(1). *or* $= \frac{1}{2}\log\lvert x^2+3x+2\rvert - \frac{3}{2}\log\left\lvert \frac{x+1}{x+2}\right\rvert$ …….(1)		

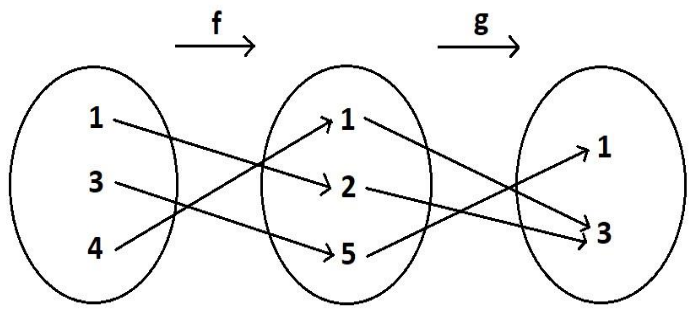
23	(i)	**a_{ij} = 3i - j**		**6**
		a_{11} 2 1 a_{12} 5 4 .... 8 7		
	(ii)	$A = a_{21}\ a_{22}$] =		
		$a_{31}\ a_{32}$		
		$A^T = [2\ 1]$ .... 3 −4	2	
		$P = \frac{1}{2}\ [A + A^T]$	½	
		1 = [4 4] 2 4 −8	1	
		= [2 2] , symmetric .... 2 −4	½	
			1	
		$Q = [\ \frac{1}{2}\ A - A^T]$		
		1 = [0 2]	½	

		2 −2 0 $= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, skew symmetric …………………………….		

		P+Q = $\begin{bmatrix} 2 & 2 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ ………………. **<u>Remark</u>** For **attempting** the question **23 (i)** , give **2 scores.**	½	

24		A.X = B 3 −2 3 *x* 8 2 1 −1 [*y*] = [1] …………………………………… 4 −3 2 *z* 4 3 −2 3 |*A*|= 2 1 −1 ………………………………………………. 4 −32 = *-17* ………………………………………………………………. 9 7 −1 −5 −1 ………………………………………………………………… *adj A* = −8 −6] ……………………………………………. −10 1 *X* = *A*$^{-1}$.B = (*A*).*B adj*	1 ½ ½ 2 1 ½ ½	6

		$\|A\|$ 1 −1 −5 −1 8 = [−8 −6 9] [1] ………………………………. −17 −10 1 7 4 1 −17 1 = [−34] = [2] ………………………………………. −17 −51 3 *x=1, y=2, z=3* **Remark** For a minimum of 7 correct entries in *adj A* , give 2 scores.		
25	(i)	$(gof)1=g(f(1))=g(2)=3$ …………………………………….. $(gof)3=g(f(3))=g(5)=1$ …………………………………….. $(gof)4=g(f(4))=g(1)=3$ ……………………………………..	½ ½ ½	

		$gof=\{(1,3),(3,1),(4,3)\}$	½+½+½	**6**
		OR  ...(1½) $gof=\{(1,3),(3,1),(4,3)\}$ ....(½+½+½)		
		$f(x_1)=f(x_2)\Rightarrow 2x_1+1=2x_2+1$	½	
		$.\Rightarrow x_1=x_2$, f is one-one	½	
		$y=2x+1$, $x=\frac{y-1}{2}\in R$, f *is onto*	1	
		$f^{-1}(x)=\frac{x-1}{2}$	1	
	(ii)			

26	(i)	Slope = $\frac{dy}{dx}$ at $x = 2$	½	
		$= 3x^2 - 1$ at $x=2$ .... =	1	
	(ii)	$3\times2^2-1 = 11$ .... $y - y_1 = m(x - x_1)$	½	
			1	
		.... $x = 2$, $y = 2^3-2=6$	½	
		.... $y-6 = 11(x-2)$	½	
		....		
	(iii)	$11x - y = 16$		
		$f^{\prime}(x) = \cos x - \sin x$	½	
		$f^{\prime}(x) = 0 \Rightarrow x = \frac{\pi}{4}$ .... $f^{\prime\prime}(x)<0$	½	
		....	½	**6**

		Maximum value = $\sin\frac{\pi}{4}+\cos\frac{\pi}{4}$ …………………………………………. $= \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\sqrt{2}$ **OR** $(\sin x+\cos x)^2=\sin^2x+\cos^2x+2\sin x\cos x$ …………………(½) $= 1+\sin 2x$ The maximum value of sin 2x = 1 $(\sin x+\cos x)^2=2$ ……………………………………………..(1) Maximum value = $\sqrt{2}$ …………………………………….(½) **Remarks** (I) For **attempting** the question **26(ii)**, give **2 scores**. (II) For direct answer, give full score for 26 (iii).	½	

27	(i)	Put cos x = t ………………………………………………		**6**
		-sin x .dx = dt ……………………………………………..	½	
		$\int sinx.\sin(cosx)\, dx = \int sint.-dt$ ………………………………	½	
		= -(-cos t) = cos t …………………	½	
		= cos(cos x)+ C ……………………..	1	
	(ii)	Put $\tan^{-1}x = t$ …………………………………………………..	½	
		$\frac{1}{1+x^2}dx = dt$ …………………………………………………	½	
		$\int_0^1 \frac{\tan^{-1}x}{1+x^2}\, dx = \int_0^{\frac{\pi}{4}} t\, dt$ …… …………………………………	½	
			½	
		$= \left[\frac{t^2}{2}\right]_0^{\frac{\pi}{4}}$ ……………………………………………………….	1	

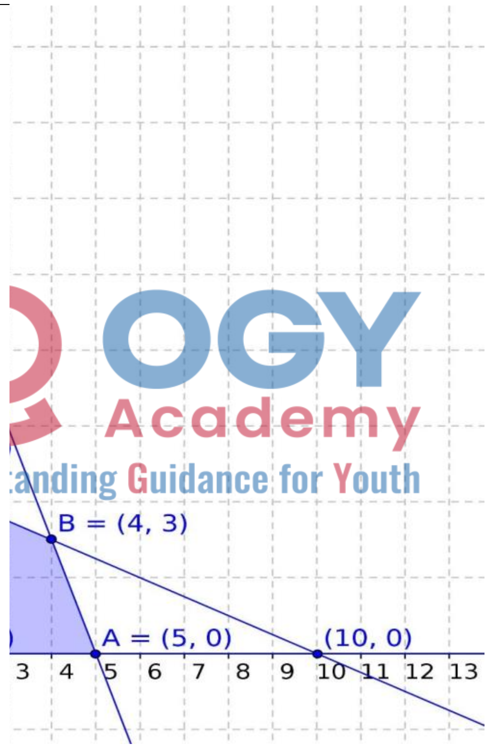
		$= \frac{\pi^2}{32}$ …………………………………………………………….	½	
28		$x + 2y = 10$	½	**6**

x	0	10
y	5	0

$3x + y = 15$

x	0	5
y	15	0

½



2½ +½

Corner Points	z = 3x+2y
O (0,0)	0
A (5,0)	15
B (4,3)	18
C (0,5)	10

½+½+½ ½

Maximum value of $z = 18$

Remarks

(I) For each correct line , give 1 score.

(II) In the figure,if only the feasible region give 2½ scores

(4,3) --------------------------

29	(i)	Area = $\int_a^b y\,dx$	½	**6**
		$= \int_1^4 \sqrt{x}\,dx$	½	
		$= \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_1^4$	1	
		$= \frac{2}{3}\left(4^{\frac{3}{2}}-1^{\frac{3}{2}}\right)$	½	
		$= \frac{2}{3}(2^3-1) = \frac{2}{3} \times 7 = \frac{14}{3}$ sq.units	½	
	(ii)	Area $= \int_a^b (f(x)-g(x))\,dx$	½	
		$= \int_0^1 \left(\sqrt{x}-x^2\right)dx$	1	
		$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{3}\right]_0^1$	1	
		$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ sq.units	½	

REMARK

ANY ALTERNATE METHOD IS ALLOWED , MARKS SHOULD BE ACCORDINGLY AND GIVE FULL SCORE FOR THE CORRECT ANSWER
