ANSWER KEY

SECOND YEAR HIGHER SECONDARY EXAMINATION March 2024

PART I/II/III

SUBJECT: MATHEMATICS (Science)

CODE NO: SY 527

VERSION: B

60 SCORES

2 HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1	i	$R = \{(1,1), (2,3), (3,5)\}$	1	
		Domain $= \{1, 2, 3\}$ Range $= \{1, 3, 5\}$	$(\frac{1}{2}+\frac{1}{2})$	
	ii	$(2,3) \in R$ but $(3,2) \notin R$ $\therefore$ R is not symmetric $\therefore$ R is not an equivalence relation [not reflexive, not transitive]	1	3
		Any other method give full mark		
2.		$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$	1	
		$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$ $7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$	$\frac{1}{2}+\frac{1}{2}$	3
		$A^2 - 5A + 7I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	1	
3	i	$\lim_{x \to 1} f(x) = \lim_{x \to 1} 2x+3 = 5$	$\frac{1}{2}$	
		$f(1) = 5$	$\frac{1}{2}$	
		$\therefore$ f is continuous at $x = 1$		
	ii)	$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^-} f(x)$	$\frac{1}{2}$	3
		$\lim_{x \to 5} kx+1 = \lim_{x \to 5} 3x-5$	1	
		$5k+1 = 15-5$ $k = 9/5$	$\frac{1}{2}$	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
4	i	$\frac{\pi}{6}$	1	
	ii	$\tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right] = \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$	1	3
		$= \tan^{-1}\left(2\times\frac{1}{2}\right) = \tan^{-1}(1) = \frac{\pi}{4}$	$\frac{1}{2}+\frac{1}{2}$	
5	i	(D) $\log x$	1	
	ii	$f'(x) = 2x-4$	$\frac{1}{2}$	
		$2x-4=0$	$\frac{1}{2}$	
		$x = 2$		3
		$(-\infty, 2)$, $f'(x) < 0$, $f(x)$ decreasing	$\frac{1}{2}$	
		$(2, \infty)$ $f'(x) > 0$ $f(x)$ increasing	$\frac{1}{2}$	
6	i	$\frac{\pi}{4}$	1	
	ii)	Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert}$	1	3
		$= \frac{(\hat{i}-\hat{j})\cdot(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}} = \frac{1-1}{\sqrt{2}} = 0$	1	
		(If $\vec{a}\cdot\vec{b} = 0$ only give $\frac{1}{2}$ score, $\lvert\vec{b}\rvert = \sqrt{2}$ give $\frac{1}{2}$ score)		

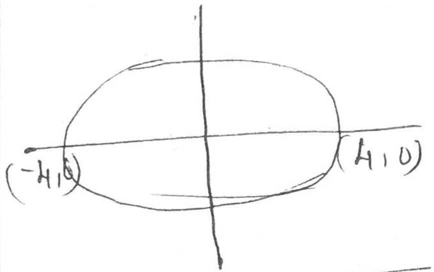
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
7		$P(A\cap B) = P(A)\ P(B) = 0.3 \times 0.4 = 0.12$	$\frac{1}{2}+\frac{1}{2}$	
		$P(A\cup B) = P(A) + P(B) - P(A\cap B)$		
		$= 0.3 + 0.4 - 0.12 = 0.58$	1	3
		$P(A/B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{0.12}{0.4} = 0.3$	1	
8		$x^5+1 = t$	$\frac{1}{2}$	
		$5x^4\,dx = dt$	$\frac{1}{2}$	
		When $x=-1$ $t=0$ When $x=1, t=2$		3
		$\int_0^2 \sqrt{t}\,dt = \frac{2}{3}\left[t^{3/2}\right]_0^2 =$	1	
		$= \frac{2}{3}\left[2^{3/2} - 0\right] = \frac{2}{3}2^{3/2}$	1	
9	i)	4	1	
	ii)	$\forall\ x_1, x_2 \in A$		
		$f(x_1) = f(x_2)$		
		$\dfrac{x_1-2}{x_1-3} = \dfrac{x_2-2}{x_2-3}$	1	
		$(x_1-2)(x_2-3) = (x_1-3)(x_2-2)$		
		$x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$		
		$-x_1 = -x_2$		
		$x_1 = x_2$ $\therefore f$ is 1-1	$\frac{1}{2}$	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		Let $y = \frac{x-2}{x-3} \Rightarrow xy - 3y = x - 2$ $\Rightarrow xy - x = 3y - 2$ $x = \frac{3y-2}{y-1} \in A$ ∴ f is onto From the above clear that $\forall y$ except 1 i.e $\forall y \in B = R - \{1\}$ $\exists x \in A$ ∴ $f^{-1}(y) = \ldots - f(x) = y$	$\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$	4
10	i	$-A$	1	
	ii	$A^T = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$ $P = \frac{A + A^T}{2} = \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$, which is symmetric $Q = \frac{A - A^T}{2} = \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$ which is skew symmetric $P + Q = A$	$\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$	4
11		Let x be the length of the square piece and $28 - x$ be the length of circular piece Perimeter of square $= x$ $4a = x \quad a = \frac{x}{4}$	1 $\frac{1}{2}$	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		Area of square $= a^2 = \frac{x^2}{16}$	$\frac{1}{2}$	
		Circumference of circle $= 28 - x$ $2\pi r = 28 - x$ $r = \frac{28-x}{2\pi}$		
		Area of circle $= \pi r^2 = \pi \frac{(28-x)^2}{4\pi^2}$ $= \frac{(28-x)^2}{4\pi}$	$\frac{1}{2}$	
		Combained Area $A = \frac{x^2}{16} + \frac{(28-x)^2}{4\pi}$		
		$\frac{dA}{dx} = \frac{2x}{16} + \frac{2(28-x)}{4\pi} \times -1$ $= \frac{x}{8} - \frac{28-x}{2\pi}$	$\frac{1}{2}$	4
		$\frac{d^2A}{dx^2} = \frac{1}{8} - \frac{1}{2\pi} \times -1$ which is +ve	$\frac{1}{2}$	
		So Area is minimum $\frac{dA}{dx} = \frac{x}{8} - \frac{28-x}{2\pi} = 0$ $\frac{x - 4(28-x)}{8\pi} = 0$ $x = \frac{112}{\pi+4}$	$\frac{1}{2}$	
		Length of square piece $= \frac{112}{\pi+4}$ Length of other piece $= 28 - \frac{112}{\pi+4}$ $= \frac{28\pi}{\pi+4}$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
12	i	Order - 2	1/2	
		degree - not defined	1/2	
	ii	$\dfrac{dy}{dx} + \dfrac{2}{x} y = x$ $\quad P = 2/x$	1	
		$I.F = e^{\int P dx} = e^{\int \frac{2}{x} dx} =$	1/2	
		$= x^2$	1/2	4
	iii	Soln: $y \times x^2 = \int x \times x^2 dx$	1	
		$y x^2 = \dfrac{x^4}{4} + c$		
13	i	$\vec{a} + \vec{b} = 2i + 3j + 4k$	1/2	
		$\vec{a} - \vec{b} = 0i - j - 2k$	1/2	
	ii	$(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b}) = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$		
		$= -2i + 4j - 2k$	2	
		Unit vector $\perp^{r}$ to $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is $= \dfrac{-2i+4j-2k}{\sqrt{4+16+4}}$	1	4
		$= \dfrac{-2i+4j-2k}{2\sqrt{6}}$		
		OR using formula $\dfrac{\vec{a} \times \vec{b}}{\lvert \vec{a} \times \vec{b} \rvert}$ give 3 score		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
14		$\vec{a_1} = \hat{i} + \hat{j}$ $\quad \vec{b_1} = 2\hat{i} - \hat{j} + \hat{k}$		
		$\vec{a_2} = 2i + j - k$ $\quad \vec{b_2} = 3i - 5j + 2k$	1	
		$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3i - j - 7k$	1	
		$a_2 - a_1 = i - k$	1/2	4
		$\lvert \vec{b_1} \times \vec{b_2} \rvert = \sqrt{59}$		
		$S.D = \left\lvert \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{\lvert \vec{b_1} \times \vec{b_2} \rvert} \right\rvert$	1	
		$= \left\lvert \frac{(i - k) \cdot (3i - j - 7k)}{\sqrt{59}} \right\rvert$	1/2	
		$= \frac{10}{\sqrt{59}}$		
15		A: getting a defective bolt		
		E_1 - Choosing machine A		
		E_2 - Choosing machine B		
		E_2 : Choosing machine C	1	
		$P(E_1) = .25$ $\quad P(E_2) = .35$ $\quad P(E_3) = .45$	1	
		$P(A/E_1) = .05$ $\quad P(A/E_2) = .04$ $\quad P(A/E_3) = .02$	1	4
		$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
16				
		$y = \frac{3}{4}\sqrt{16-x^2}$	$1\frac{1}{2}$	
		$\therefore A = 4\left\lvert \int_0^4 y\,dx \right\rvert$	1	
		$= 4\left\lvert \frac{3}{4}\int_0^4 \sqrt{4^2-x^2} \right\rvert dx$		4
		$= 3\left[\frac{x}{2}\sqrt{4^2-x^2} + \frac{4^2}{2}\sin^{-1}\frac{x}{4}\right]_0^4$	1	
		$= 3\left[\sin^{-1} 1 - 0\right] = 3\times\frac{8\pi}{2}$		
		$= 12\pi$ Sq. unit	$\frac{1}{2}$	
		Remark: Direct answer give 1 score		
17		$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$	1	
		$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$		
		$\lvert A\rvert = 2\times 0 + 3\times -2 + 5 = -1$	1	
		$\therefore A^{-1}$ exist		
		Cofactor matrix of A $= \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$A^{-1} = \dfrac{adj\,A}{\lvert A\rvert} = \dfrac{\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}}{-1}$	1	6
		$X = A^{-1}B = -\dfrac{1}{1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$	1	
		$= -\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$	1	
18	i	$\cos x - \sin y \dfrac{dy}{dx} = x\dfrac{dy}{dx} + y$	1	
		$\dfrac{dy}{dx} = \dfrac{\cos x - y}{x + \sin y}$	1	
	ii	$\dfrac{dx}{dt} = 3a\cos^2 t \times -\sin t$	1/2	
		$\dfrac{dy}{dt} = 3a\sin^2 t \cos t$	1/2	
		$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = -\dfrac{3a\cos^2 t \sin t}{3a\sin^2 t\cos t}$	1	
		$= \dfrac{-\sin t}{\cos t} = -\tan t$		
	iii	$\dfrac{dy}{dx} = 2\sin^{-1}x \times \dfrac{1}{\sqrt{1-x^2}}$	1	
		$\sqrt{1-x^2}\dfrac{dy}{dx} = 2\sin^{-1}x$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		Diff; $\sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx} \times \frac{1}{2\sqrt{1-x^2}} \times -2x = 2\times\frac{1}{\sqrt{1-x^2}}$ $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$	1	6
19	i	$\frac{x-1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$	1	
		$x-1 = A(x-3) + B(x-2)$ $x=2, \; A=-1$ $x=3, \; B=2$	1	
		$\int \frac{x-1}{(x-2)(x-3)} = \int \frac{-1}{x-2}dx + \int \frac{2}{x-3}dx$	1/2	
		$= -\log\lvert x-2\rvert + 2\log\lvert x-3\rvert$	1/2	
	ii	$I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx = \int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$	1	
		$= \int_0^{\frac{\pi}{4}} \log\left(1 + \frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}\tan x}\right)dx$		
		$= \int_0^{\frac{\pi}{4}} \log\left(\frac{1+\tan x + 1 - \tan x}{1+\tan x}\right)dx$	1/2	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score

$$\int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan x}\right) dx = \int_0^{\frac{\pi}{4}} (\log 2 - \log(1+\tan x))\, dx$$

$$= \int_0^{\frac{\pi}{4}} \log 2\, dx - \int_0^{\frac{\pi}{4}} \log(1+\tan x)\, dx$$

$$= \int_0^{\frac{\pi}{4}} \log 2\, dx - I$$ — Score: 1

$$2I = \int_0^{\frac{\pi}{4}} \log 2\, dx = \frac{\pi}{4} \log 2$$ — Total Score: 6

$$I = \frac{\pi}{8} \log 2$$ — Score: 1/2

$$\int_0^{\frac{\pi}{4}} \log(1+\tan x)\, dx = \frac{\pi}{8} \log 2$$

Remark: Try to answer using the property give 3 score.

20

20.

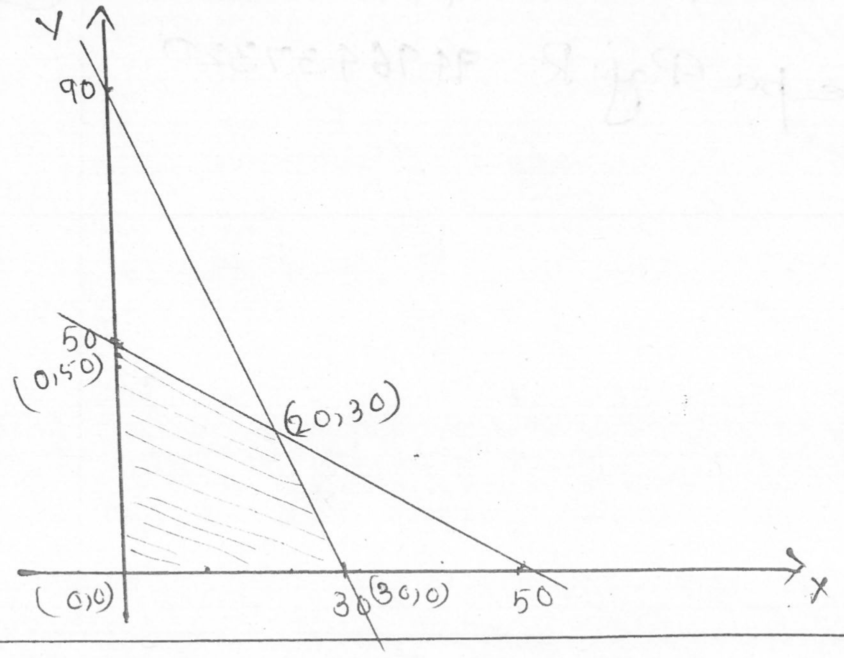
$x + y \leq 50$

x	0	50
y	50	0

$3x + y \leq 90$

x	0	30
y	90	0

Score: 1



Score: 3

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$Z=60x+15y$		
		Corner points: (0,0), (30,0), (0,50), (20,30); $Z=60x+15y$: 0, 1800, 750, 1650	2	6
		Maximum value occurs at (30,0)		
		The solution is $Z=1800$ at $(30,0)$		
		Corner points | $Z=60x+15y$		

Corner points	$Z=60x+15y$
(0,0)	0
(30,0)	1800
(0,50)	750
(20,30)	1650

1\. RANJU.M.KUM