SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2024

PART III

SUBJECT: STATISTICS

CODE NO: **SY 532** VERSION:

SCORES: **60** HOURS: **2**

Qn, No.	Sub Qns	Answer Key / Value Points	Score	Total Score
1		(b) 1	1	1
2		(c) Continuous	1	1
3		(d) F – statistic	1	1
4		(a) θ	1	1
5		(c) Power of a test	1	1
6		(d) F	1	1
7		(b) Seasonal Variation	1	1
8		Explanation of positive correlation/scatter diagram/example Explanation of negative correlation/scatter diagram/example	1 1	2
9		We have, $b_{yx} = r\frac{\sigma_y}{\sigma_x}$ $ie,\ 0.23 = 0.45 \times \frac{\sigma_y}{10}$ $\therefore \sigma_y = \frac{0.23 \times 10}{0.45} = 5.11$	1 ½ ½	2
10		$y = x^2 + 3x + 4$ $\frac{dy}{dx} = 2x + 3 \times 1 = 2x + 3$	2	2
11		$\int_0^1 x^2 dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$	1 + 1	2
12		The number of heads follows a Binomial distribution with $n = 16, p = 1/2$ Mean $= np = 16 \times 1/2 = 8$ Variance $= npq = 16 \times 1/2 \times 1/2 = 4$ ***(Identifying the problem as binomial, give 1 score)***	 1 1	2
13		Any 4 properties of Normal distribution (½ score each)	$4 \times ½ = 2$	2

<table>
<tr><td>14</td><td></td><td><table><tr><th>Sl No</th><th>Sample</th><th>Sample mean</th></tr><tr><td>1</td><td>2, 3</td><td>2.5</td></tr><tr><td>2</td><td>2, 5</td><td>3.5</td></tr><tr><td>3</td><td>3, 5</td><td>4</td></tr><tr><td></td><td>Total</td><td>10</td></tr></table>Mean of sample means $=\frac{10}{3}=3.33$</td><td>½ + ½
1</td><td>2</td></tr>
<tr><td>15</td><td></td><td>Unbiasedness, Consistency, Efficiency, Sufficiency (½ score each)</td><td>$4\times ½=2$</td><td>2</td></tr>
<tr><td>16</td><td></td><td>Explanation of assignable causes/Example
Explanation of chance causes/Example</td><td>1
1</td><td>2</td></tr>
<tr><td>17</td><td></td><td>$CL=\overline{\overline{x}}=\frac{\sum\overline{x}}{m}=\frac{514.8}{25}=20.592,\ \overline{R}=\frac{\sum R}{m}=\frac{120}{25}=4.8$
$LCL=\overline{\overline{x}}-A_2\overline{R}=20.592-0.577\times4.8=17.8224$
$UCL=\overline{\overline{x}}+A_2\overline{R}=20.592+0.577\times4.8=23.3616$
<i>(For any 2 correct formulae, give 1 score)</i></td><td>½ + ½
½
½</td><td>2</td></tr>
<tr><td>18</td><td></td><td>Secular Trend, Seasonal variation, Cyclic variation, Irregular variation (½ score each) OR
Secular Trend, Periodic movements, Irregular Variations – Give full score.</td><td>$4\times ½=2$</td><td>2</td></tr>
<tr><td>19</td><td></td><td>$\sum p_1=182,\ \sum p_0=156$
Simple aggregate price index number $=\frac{\sum p_1}{\sum p_0}\times100=\frac{182}{156}\times100=116.67$</td><td>½ + ½
½ + ½</td><td>2</td></tr>
<tr><td>20.</td><td>(a)</td><td>$\overline{x}=60,\ \overline{y}=100,\ \sigma_x=20,\ \sigma_y=15,\ r=-0.81$
$b_{yx}=r\times\frac{\sigma_y}{\sigma_x}=-0.81\times\frac{15}{20}=-0.6075$
Regression line of Y on X is
$y-\overline{y}=b_{yx}\left(x-\overline{x}\right)$
$ie,\ y-100=-0.6075\left(x-60\right)$
$ie,\ y=-0.6075x+136.45$ (<i>This simplification is not compulsory</i>)</td><td>½ + ½
1
1</td><td rowspan="2">4</td></tr>
<tr><td></td><td>(b)</td><td>When x = 70, $y-100=-0.6075\times\left(70-60\right)$
$\therefore y=93.925$</td><td>½
½</td></tr>
<tr><td>21</td><td>(a)</td><td>$E(X)=\sum xp(x)\quad=-1\times0.4+0\times0.3+1\times0.3=\underline{\underline{-0.1}}$</td><td>½ + ½</td><td rowspan="2">4</td></tr>
<tr><td></td><td>(b)</td><td>$E(X^2)=\sum x^2p(x)\quad=\left(-1\right)^2\times0.4+0^2\times0.3+1^2\times0.3=0.7$
$V(X)=E\left(X^2\right)-\left(E(X)\right)^2$
$=0.7-\left(-0.1\right)^2\quad=0.7-0.01=0.69$</td><td>½ + ½
1
½ + ½</td></tr>
</table>

22

$$p(x)=\frac{\lambda^{x}e^{-\lambda}}{x!},\quad x=0,1,2,.. \qquad ie,\ p(x)=\frac{0.1^{x}e^{-0.1}}{x!},x=0,1,2,..$$

(1) $P(X=2)=\frac{0.1^{2}e^{-0.1}}{2!}=\frac{0.01\times e^{-0.1}}{2}=0.005\times e^{-0.1}$ — 1

$=0.005\times 0.9048=\underline{\underline{0.004524}}$ (*Simplification not compulsory*)

(2) $P(X\geq 2)=1-P(X\leq 1)=1-\left[P(x=0)+P(X=1)\right]$ — 1

$=1-\left[\frac{0.1^{0}e^{-0.1}}{0!}+\frac{0.1^{1}e^{-0.1}}{1!}\right]$ — 2

$=1-1.1\times 0.9048=\underline{\underline{0.00472}}$ (*Simplification not compulsory*)

(Give 1 score for writing the pmf of Poisson distribution only)

Total: 4

23

Let X be a normal variable with $\mu=40$ and $\sigma=10$

$P(30<X<50)=P\left(\frac{30-40}{10}<Z<\frac{50-40}{10}\right)$ — 1

$=P(-1<Z<1)=2\times P(0<Z<1)$ — 1

$=2\times 0.3413=0.6826$ — 1

No. of students got marks between 30 and 40 = $600\times 0.6826=410$ (*approx*) — 1

Total: 4

24

Concept of statistic	Example of statistic	1 + 1
Concept of parameter	Example of parameter	1 + 1

Total: 4

25

We have to test $H_0:\mu=50$ against $H_1:\mu>50$ — 1

The test statistics is, $Z=\frac{\overline{x}-\mu}{s/\sqrt{n}}\sim N(0,1)$ — 1

$=\frac{52-50}{3/\sqrt{100}}=\frac{2\times 10}{3}=6.67$ — 1

The critical region is $|Z|>Z_{\alpha}$. or $Z>Z_{\alpha}$ Here $|Z|=6.67>Z_{\alpha}=2.33$. — ½ + ½

(calculated value > table value). We reject. So the mean is greater than 50.

(Inference is not compulsory)

Total: 4

26

Source	Df	Sum of squares	Mean sum of squares	F
Between samples	5	**60**	12	**3**
Within samples	**19**	76	**4**	
Total	24	**136**		

Score for completing the table: 2½

The critical region is $F>F_{\alpha}$ — ½

Here F = 3 < $F_{0.01}$ = 4.17. So we accept the null hypothesis at 1% significance level. ***(Give 3 scores if the table only is completed)*** — 1

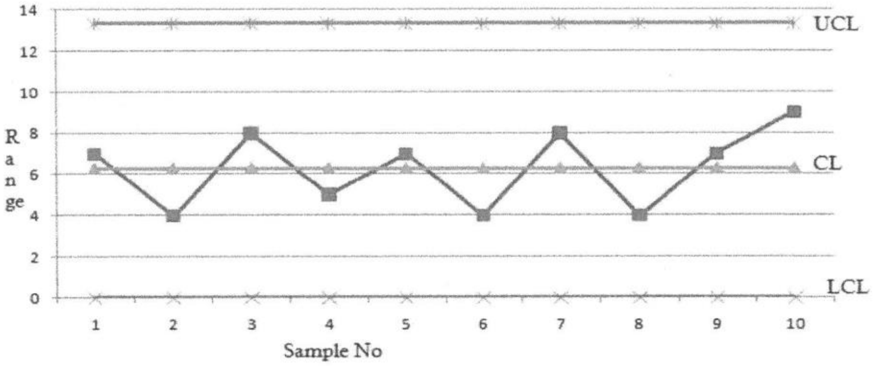
Total: 4

27

$$CL = \overline{R} = \frac{\sum R}{m} = \frac{63}{10} = 6.3$$ 1

$$LCL = D_3\overline{R} = 0$$ 1

$$UCL = D_4\overline{R} = 2.115 \times 6.3 = 13.3245$$ 1



1

4

Here the process is in control.***(Also consider proper explanation without graph) (Give 1 score for rough sketch of R chart)***

28

$$r = \frac{n\sum XY - \sum X \sum Y}{\sqrt{n\sum X^2 - \left(\sum X\right)^2} \times \sqrt{n\sum Y^2 - \left(\sum Y\right)^2}}$$ 1

X	Y	X^2	Y^2	XY
5	1	25	1	5
10	6	100	36	60
5	2	25	4	10
11	8	121	64	88
12	5	144	25	60
4	1	16	1	4
3	4	9	16	12
2	6	4	36	12
7	5	49	25	35
1	2	1	4	2
$\sum X = 60$	$\sum Y = 40$	$\sum X^2 = 494$	$\sum Y^2 = 212$	$\sum XY = 288$

2

5

$$\therefore r = \frac{10 \times 288 - 60 \times 40}{\sqrt{10 \times 494 - (60)^2} \times \sqrt{10 \times 212 - (40)^2}} = \frac{480}{\sqrt{1340} \times \sqrt{520}}$$ ½+½

$$= \frac{480}{834.745} = 0.575$$ ½

The correlation is moderate positive or positive. ½

(Also consider the formula $r = \frac{Cov(x,y)}{\sigma_x \times \sigma_y}$

$Cov(x,y) = 4.8$, $\sigma_x = 3.66$, $\sigma_y = 2.28$ and $r = 0.575$)

29 (a) (ii) Chronologically — 1

(b)

Year	Sales	4 Yearly moving total	4 yearly moving average	moving average centered
2005	128			
2006	265			
		1146	286.5	
2007	341			331.125
		1503	375.75	
2008	412			409
		1769	442.25	
2009	485			471.875
		2006	501.5	
2010	531			527.5
		2214	553.5	
2011	578			
2012	620			

(Give 3 score for 4 yearly moving average without centered values)

4 — 5

30

p_0	q_0	p_1	q_1	p_0q_0	p_1q_0	p_0q_1	p_1q_1
9.25	5	15	5	46.25	75	46.25	75
8	10	12	11	80	120	88	132
4	6	5	6	24	30	24	30
1	4	1.25	8	4	5	8	10
Total				**154.25**	**230**	**166.25**	**247**

2

(a) Laspeyre's Index No.= $\dfrac{\sum p_1q_0}{\sum p_0q_0}\times 100 = \dfrac{230}{154.25}\times 100 = 149.11$ — 1

(b) Paasche's Index No. $= \dfrac{\sum p_1q_1}{\sum p_0q_1}\times 100 = \dfrac{247}{166.25}\times 100 = 148.57$ — 1

(c) Fisher's Index No. $= \sqrt{L\times P} = \sqrt{149.11\times 148.57} = 148.84$ — 1

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